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Trajectory Design using State Feedback for Satellite on a Halo Orbit

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ABSTRACT

A simple state feedback method for satellite trajectory design on a halo orbit is developed. The communication link using satellite trajectory control is from the earth to the satellite and then to the far side of the moon. The nonlinearities inherent to the halo orbit problem are treated as trajectory-dependent, persistent disturbance inputs. A controller has been designed along with a full order estimator of linear state variable feedback using pole placement method. This type of compensated linear controllers gives satisfactory performance for limited dynamic range and limited input. The design has been made by specially constructed programs and the results have been checked up using MATLAB tools.

Key words: Satellite trajectory control, Pole placement method, Halo orbit, State variable feedback

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INTRODUCTION

Satellite is placed into orbit around the Earth, other planets, or the Sun. It has come into use very recently. Now-a-days, artificial satellites play key roles in communication industries, in military intelligence, and in scientific study of both Earth and outer space. For direct access from the earth controller section by the space, the nearby lunar operation is processed. At far side lunar operations for the earth moon space communication it is very difficult until an uninterrupted link is established. With the help of adhoc network communication technique, a relay satellite is launched to follow the uninterrupted link between the earth and the space satellite system. This relay satellite follows the orbit that would provide continuous communication coverage for most of the moon's far side. In 1966 this concept was developed for eliminating the complexity at far–side communication between earth-moon space satellites. This so-called orbit actually noted is halo orbit. Halo orbits are large three-dimensional orbits shaped like the edges of a potato chip. The y-amplitude of the Genesis halo orbit extends from the x-axis to the maximum y-value of the orbit. The computation of halo orbits follows standard nonlinear trajectory computation algorithms based on parallel shooting. The paper considers the satellite trajectory control in the Earth- Moon orbital system. The objective is to determine an appropriate method for the stabilization of a spacecraft to the halo orbit, while simultaneously stabilizing the attitude of the spacecraft to stay inertial fixed. Halo orbits follow unstable limit cycles centred on the collinear Lagrangian points that are unforced solutions to the restricted 3-body problem. The halo orbit is particularly interesting because Lagrangian points are behind the Moon, and because it is the point with the lowest gravitational potential energy needed to escape the Earth-Moon system. A satellite or space station following a sufficiently large halo orbit trajectory can facilitate communication between Earth and the far side of the Moon, and also serve as a launch pad for far away space missions. Using state feedback, we are able to develop control schemes that stabilize these trajectories.

PRIVILEGES OF STATE FEEDBACK OVER OUTPUT FEEDBACK

Output feedback is sufficient for many systems; state feedback is very useful for MIMO (multi-input multi-output) systems and for control systems with optimal constraints such as those requiring minimal control effort or minimum time to final value. The response for one particular set of gains is shown in the scope plot in Fig. 2. Inclusion of velocity feedback adds damping to the system (reduces overshoot) and speeds up the system response (reduces settling time).

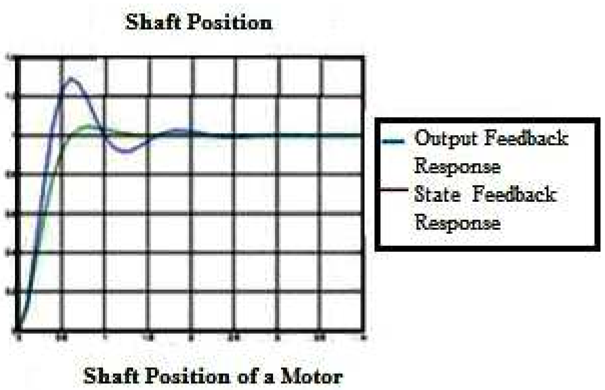

Fig. 1 Shaft position of meter

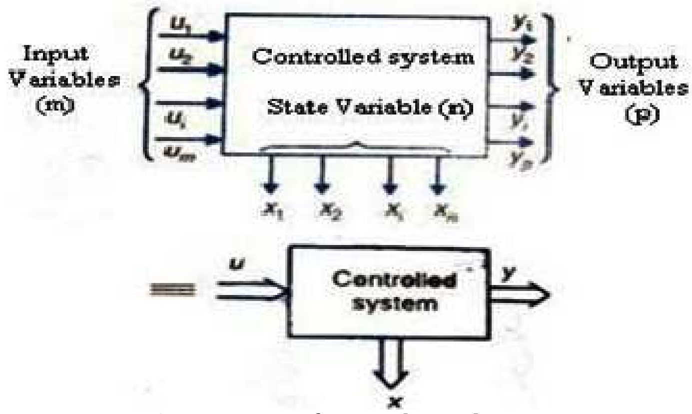

Fig. 2 Structures of a general control system

POLE PLACEMENT USING STATE FEEDBACK

The state equation for state x with input u is given by a relation as:

$$dx(t)/dt=Ax(t)+Bu(t) \tag{1}$$

$$y(t) =Cx(t)+Du(t) \tag{2}$$

In the above state space equation A is called system matrix, B the input coupling, C the output coupling matrix and D input- output coupling or direct transmission matrix. The controllability matrix is then defined as

$$[B\ AB\ A^2B\ \ldots A^{n-1}B],$$

where n is the system order or equivalently the number of states. The theorem concerning pole placement is then introduced.

The system is controllable (i.e., the closed-loop poles can be placed in any desired position) if the determinant of the controllability matrix

$$[B\ AB\ A^2B\ \ldots A^{n-1}B]$$

is non-zero. The controllability model for a system is then introduced. It is shown that for any given set of desired poles, a set of feedback gains can be derived to place the system closed-loop poles at the desired positions. Finally, Ackermann's formula is introduced to compute the state-feedback gains to place the poles in the desirable positions.

Satellite Trajectory Control

We want to design a model of state feedback system to keep the satellite on a halo orbit trajectory. The translunar satellite halo orbit is shown in Fig.3

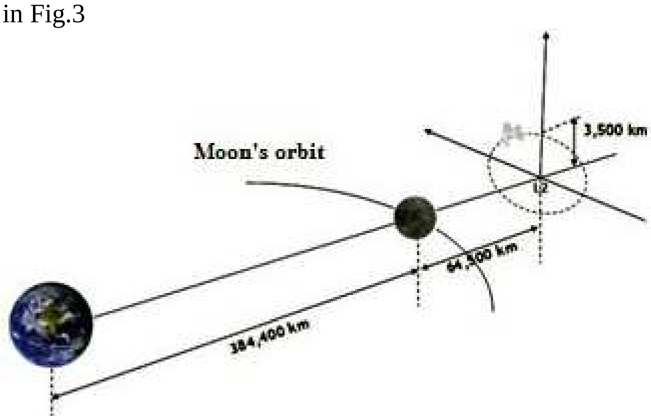

Fig.3 The translunar satellite halo orbit

The linearized (and normalized) equations of motion of the satellite around the translunar equilibrium points are given by the equation [3]:

$$\bar{x} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{pmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_3 \tag{3}$$

The state vector x yields the satellite position and velocity, and the inputs, u_i for i= 1,2,3 are the engine thrust accelerations in the ξ, η and ζ direction respectively. First, we check whether the translunar equilibrium point is stable location or not. The problem is to design a controller that commands the satellite thrusters in such a manner that the actual orbit remains near the desired orbit. Before commencing with the design, we investigate controllability independently.

We also check system controllability for u_1, u_2 and u_3 by using MATLAB.

For input u_1:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix}$$

$B = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$

$C = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

$D = [0]$

Check for controllability of the system for u_1

Compute controllability matrix

$$Pc = \begin{bmatrix} 0 & 1 & 0 & 3.3809 & 0 & 20.1921 \\ 0 & 0 & -2.0 & 0 & -2.3810 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 3.3809 & 0 & 20.1921 & 0 \\ 0 & -2 & 0 & -2.3810 & 0 & -35.1688 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

n=det(Pc)

n= determinant of controllability matrix

n=0

The system is not completely controllable for $u_{1.}$

For input u_2:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix}$$

$B = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$

$C = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

$D = [0]$

Check for controllability of the system for u_1

Compute controllability matrix

$$Pc = \begin{bmatrix} 0 & 0 & 2 & 0 & 2.3810 & 0 \\ 0 & 1 & 0 & -6.1904 & 0 & 8.7975 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2.3810 & 0 & 35.1688 \\ 1 & 0 & -6.1904 & 0 & 8.7975 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

n=det(Pc)

n=0

The system is not controllable for u_2

For input u_3:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix}$$

B =[0 0 0 0 0 1]
C =[0 1 0 0 0 0]
D =[0]

Check for controllability of the system for u_1

Pc=ctrb(A,B);
n=det(Pc)
n=0

The system is not controllable for u3.

Suppose that we can observe the position in the η direction. Then we determine the transfer function from u_2 to η.

Let y =[0 1 0 0 0 0]x

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix}$$

B =[0 0 0 0 1 0]
C =[0 1 0 0 0 0]
D =[0]

```
[num,den]=ss2tf(A,B,C,D);
printsys(num,den)
```

num/den =
1 s^4 + 5.7239e-016 s^3 - 4.1905 s^2 + 1.8262e-015 s - 23.548
----
s^6 + 4.4409e-016 s^5 + 1.9999 s^4 + 2.9575e-015 s^3 - 19.9653 s^2 + 4.9155e-015 s - 51.5796

For u_2 to η

Transfer function $T(s) = \frac{s^4 - 4.1905\, s^2 - 23.548}{s^6 + 2\, s^4 - 19.9653\, s^2 - 51.5796}$

T(s) can reduced by eliminating common factor are (s^2+3.1834)

The reduced transfer function is

$$sys\_tf = T(s) = \frac{s^2 - 7.3815}{s^4 - 1.1837\, s^2 - 16.2030}$$

The state space representation of the transfer function is given bellow:

[A,B,C,D]= tf2ss(n,d)

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix}$$

B =[1 0 0 0]
C=[0 1.0000 0 −7.3815]
D =[0]

Check for controllability of the system

Pc=ctrb(A,B);
n=det(Pc)
n=1

The system is controllable
Using state feedback $u_2 = -Kx$
We calculate the gain matrix K which places the desired poles (using Ackermann's formula)

```
>> p=[-1+i;-1-i;-10;-10]
>> k=acker(A,B,p)
```

k = 22.0000 143.1837 240.0000 216.2030

Simulation of the System Response to Check Specifications

The system response can be generated using the MATLAB commands step or lsim; but for better understanding of the feedback system, this can be done by building it in SIMULINK using the State Variable block. The block diagram for Satellite Trajectory Control system and the system response are shown in Fig. 4.

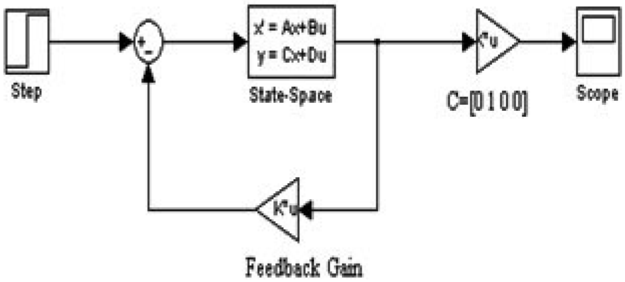

Fig. 4 (a) State Feedback Controller for halo orbit trajectory control

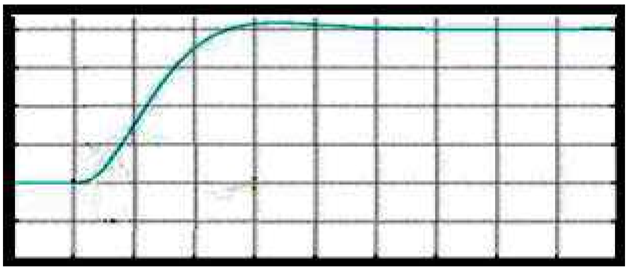

Fig. 4 (b)

In our proposed work we check the closed-loop pole reinforces the validity of Ackermann's formula and the pole-placement technique. This is a useful "replacement" of the proofs included in a more rigorous course in state-feedback control.

DESIGNING OBSERVERS FOR STATE ESTIMATION

State estimates can provide valuable information about important variables in a physical process, for example feed composition to a reactor, environmental forces acting on a ship, load torques acting on a motor, etc. In this case, the actual state is replaced by an estimate of that state derived from a state estimator or a state observer. Here shown a step-by-step procedure for designing a state estimator. An example of state estimation is to control the pitch angle of helicopter. The performance specifications for the controller are the same as those outlined in the previous section. The initial conditions for the estimator are all assumed to be zero. The state-feedback controller and state estimator are built using SIMULINK. The system block diagram and the system response are shown in Fig. 5.

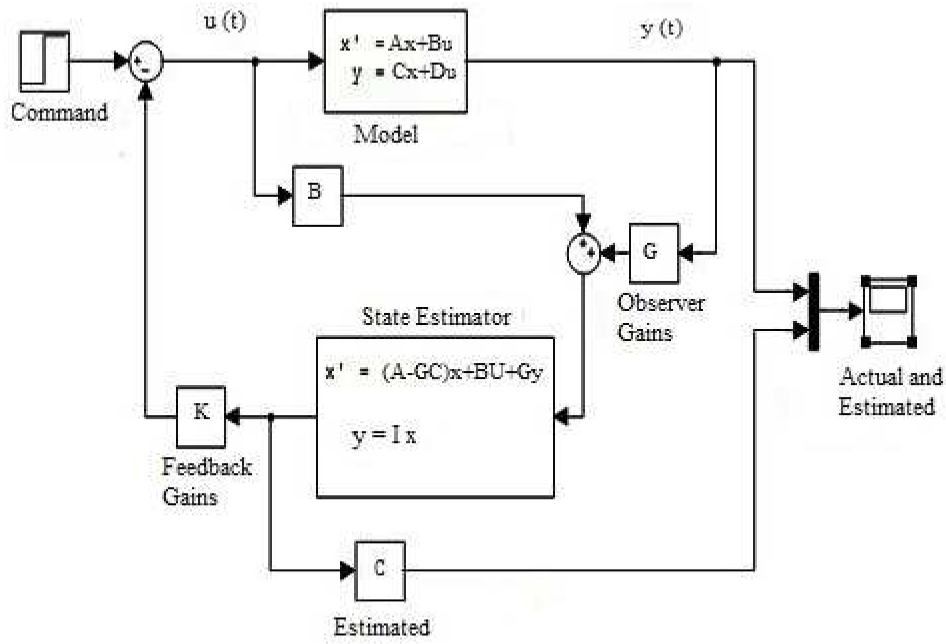

Fig. 5

CONCLUSION

In this paper, we proposed a design method of an adaptive tracking controller to keep the satellite on a halo orbit trajectory that can be seen from the earth so that lines of communication are accessible at all times.. We first present a system test for controllability and observability and proceed to describe one procedure for determining an optimal control system. Using the powerful notion of state variable feedback, we introduce the pole placement design technique. Ackermann's formula can be used to determine the state variable feedback gain matrix to place the system poles at the desired locations. The closed loop system pole locations can be arbitrarily placed, if and only if the system is controllable. The state-feedback controller and state estimator are built using SIMULINK.

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