Available online www.ejaet.com

European Journal of Advances in Engineering and Technology, 2015, 2(5): 117-122

Research Article ISSN: 2394 - 658X

Parameter Estimation in Surface Radiation Heat Transfer using Least Square Residual Method and Bayesian Approach in Vertical Plate

Anu Nair P and K Karuppasamy

Department of Mechanical Engineering, Regional Centre Anna University, Tirunelveli, India
anunair67@gmail.com

ABSTRACT

Emissivity is a parameter which helps to determine emissive power of a Surface. Much experimental work in radiation has been done to find out the value of emissivity for different surfaces. The paper reports the ability of Bayesian Approach and least square method in parameter estimation when radiation heat transfer is encountered. The aim is to estimate the value of Emissivity of material using Bayesian optimization method and least square residual method where air is used as a working medium. The steady temperature time history is mainly used to estimate Emissivity of material in Bayesian and least square optimization method. The experimental setup has been designed and fabricated. Finally, Estimated Parameter is compared with Actual benchmark.

Key words: Radiation Heat transfer, vertical plate, least square residual method, Bayesian approach

INTRODUCTION

Everybody at a temperature greater than zero kelvin emit thermal radiations of all wavelength within a certain range. The energy emitted depends on (1) the temperature of the body and (2) nature of radiating surface of the body. All states of matter emit radiation in the form of electromagnetic waves due to thermal excitement of its composing molecules. The important characteristics of radiation heat transfer are it increases non linearly with increasing temperature, it travel along straight lines with the speed of light, Radiation require no material medium to propagate and At low temperature, radiation may be significant.

Elenbass [1] conducted experimental work in laminar natural convection heat transfer in smooth parallel plate vertical channel was investigated and reported a detailed study of the thermal characteristics of cooling by natural convection. Fishenden and Saunders [2] did experimental work in natural convection heat transfer work on upward facing horizontal surface where square plate placed in air. Bosworth [3] who gave very few information on the working fluid and plate shape. Mikheyev [4] did experiment in rectangular plates. Mikheyev found that in same conditions, heat transfer rate from a vertical plate was 70% less than that of horizontal plate.

Nada [5] experimentally conducted velocity field measurements for natural convection heat transfer flow in asymmetrically and symmetrically heated vertical channels. in the asymmetrically heated channel, one plate was kept above the atmospheric temperature and the other one below it where in the In symmetrically heated channel, both plates are heated above the atmospheric temperature. Velocity measurements were done for two Rayleigh numbers (Ra = 2×10^6 and 4×10^6) both in the turbulent layer region. Oztop et al [6] conducted numerical investigation of natural convection heat transfer and fluid flow of two heated partitions within a square enclosure. The left wall and top wall were at Isothermal while the bottom and the right side wall were at adiabatic in nature. The two heated partition were placed at the bottom of the square enclosure at different aspect ratio and looked into the effect of heights and position of heated partition. The study were done at different Rayleigh number in the range of 10^4 to 10^6.The energy and flow Equations were solved with TDMA using finite difference Equation based on the finite control volume approach with non -staggered grid arrangement and SIMPLEM algorithm. Krishnan and Balaji [7] conducted a synergistic approach to parameter estimation in multimode heat transfer. This paper reports the efficacy of the least square residual method in parameter estimation when more than one mode of heat transfer is encountered. From literature review it can be found that free convection in vertical channel geometry with discrete heat source has considerable attention. The goal of the study is to estimate the emissivity of Stainless steel plate using least square residual method and Bayesian Approach where test fluid was air and compare it with actual values.

EXPERIMENTAL METHODOLOGY

As a first step, a experiments has been designed by studying the heat transfer in an isolated heated vertical plate. The details of the experimental setup are shown in Table 1.

Table- 1 Details of Experimental Set-up

S No	Description	Dimension/Range
1	Rectangular Vertical SS Plate (1 no.)	250×50× 3mm
3	Heat Input	0-300 W
4	Single Phase Closed Type Dimmerstat	0-1 Amps
5	Ammeter	0-1 Amps
6	Multimeter	350 V_{AC}
7	K-type Thermocouple with indicator	0-1000°C

Fig .1 Photographic view of the experimental setup

Fig. 2 Adiabatic plate assembly

Experimental apparatus has been specially planned and formulated to carry out investigations on electronics devices. The apparatus consists of a stainless steel plate and flat heater fitted on a support structure in a vertical fashion. The photographic view of the experimental set up is shown in Fig. 1.The geometry considered in the present study is a heated plate kept in between two adiabatic side plates, all parallel and vertical, with side plates symmetrically spaced from the central plate. The heat input to the heater is measured by an ammeter and a multimeter and is varied by a dimmerstat. Ammeter is connected in series manner and multimeter are connected in Parallel manner. . The side adiabatic plate assembly is fixed to the slotted L-angle. The entire setup is placed inside large rectangular enclosure which opens at the top and bottom to isolate experiment from external disturbances. Temperatures are measured using K-type thermocouple. The central heat source is hung vertically using Teflon rods through thin metal strips. Teflon wire used has 10mm diameter and 5m length. The Teflon rods and the metal strips serve the purpose of minimizing the conduction losses. The Teflon rods pass through holes in the slotted L-Angle, above which there is a hexagonal nut, which facilitate the adjustment of the position of the central plate so as to align it with respect to the side plate. Side adiabatic plate assembly consists of Stainless steel plate, non rubberised cork and wooden box containing glass wool. It is as shown in Fig.2.The side stainless steel plates are of dimension 250 x250 x 3mm and highly polished to obtain the lower emissivity. As in the case of central plate, blind holes of 1.5 mm width and 1.5 mm deep, are drilled at the points of temperature measurement, into which thermocouples are fixed. Six points, three on each plate are chosen for measurement of temperature. The non - rubberized cork sheet of one inch thickness and having the thermal conductivity of 0.044 W/mK forms the first layer of insulation. The plates are fixed to wooden boxes by inserting non rubberized cork in between them in order to minimize conduction of heat from the stainless steel plate to the wooden box. The wooden box is filled with the glass wool with small air gap to simulate adiabatic condition. Glass wool insulation has very low thermal conductivity 0.04 W/mK. Each of the adiabatic plate assembly is fixed to slotted L-Angle. The parameter varied during the experimentation is heat input to the heater and height ratio.

Procedure for the least Square Residual Optimization Method and Bayesian Approach

The experimentation consists of plate heater assembly which was given definite amount of power supply that was switched off after steady state was reached. Temperature measurements were done at regular interval of fifty seconds. The experiment was carried out under controlled condition; simulating natural convection .The experiment was done for several power supplies.

In this method, typically all the properties of the system will be known beforehand or a priori and the temperature response of the system will be sought. However, in this case the experimentally measured temperature response is available and to estimate or retrieve the value of ε.

RESULTS AND DISCUSSION

LEAST SQUARE RESIDUAL METHOD

The standard empirical correlation for vertical plate losing heat by surface radiation is given by $Q = -\varepsilon A(T^4 - T_a^4)$. The constant ε in the above relation was determined by using least square residual method and Bayesian Approach. Fig.3 shows the Variation of Plate temperature with Time. Time increases with increase in plate temperature due to the lattice vibration of molecules between plate and heater.

Here Q = Heat transfer by Radiation, T_a =Ambient Temperature, T =Temperature of the SS plate, A = Surface Area and T_i = Instantaneous Temperature

Assumptions

- There is no heat loss from/to the stainless steel plate other than that due radiation from/to the wall of the enclosure
- The temperature of the enclosure remains constant throughout.
- The properties of the SS plate do not change with temperature
- The foil is spatially isothermal(lumped capacitance formulation)
- For the above assumptions, the heating of the SS Plate can be mathematically represented as

$$mc_p \frac{dT}{dt} = -\varepsilon A(T^4 - T_a^4) \quad (1)$$

Eqn (1), the left hand side represent the rate of change of enthalpy and the right hand side represents the heat transfer by Radiation. Rearranging Eqn (1) and it becomes

$$\frac{dT}{T^4 - T_a^4} = \frac{-\varepsilon\sigma A dt}{mC_p}$$

The LHS can be integrated using Partial Fraction

$$\int_{T_i}^{T} \frac{dT}{T^4 - T_a^4} = \int_{T_i}^{T} \frac{dT}{\left(T^2 - T_a^2\right)\left(T^2 + T_a^2\right)}$$

$$= \frac{1}{2T_a^2}\int_{T_i}^{T}\left(\frac{1}{(T^2 - T_a^2)} - \frac{1}{(T^2 + T_a^2)}\right).dT$$

$$= \frac{1}{2T_a^2}\int_{T_i}^{T}\left[\left\{\frac{1}{2T_a}\left(\frac{1}{(T - T_a)} - \frac{1}{(T + T_a)}\right)\right\} - \frac{1}{T^2 - T_a^2}\right].dT$$

$$= \frac{1}{4T_a^3}\int_{T_i}^{T}\frac{dT}{(T - T_a)} - \frac{1}{4T_a}\int_{T_i}^{T}\frac{dT}{(T + T_a)} - \frac{1}{2T_a^2}\int_{T_i}^{T}\frac{dT}{(T^2 + T_a^2)}$$

$$= \frac{1}{4T_a^3}\ln\left(\frac{T - T_a}{T + T_a}\right)_{T_i}^{T} - \frac{1}{2T_a^2}\frac{1}{T_a}\tan^{-1}\left(\frac{T}{T_a}\right)_{T_i}^{T}$$

$$= \frac{1}{4T_a^3}\left(\ln\left(\frac{T - T_a}{T + T_a}\right) - \ln\left(\frac{T_i - T_a}{T_i + T_a}\right) - 2\tan^{-1}\left(\frac{T}{T_a}\right) + 2\tan^{-1}\left(\frac{T_i}{T_a}\right)\right) = \frac{-\varepsilon\sigma At}{mC_p}$$

The RHS can be integrated as

$$\frac{1}{4T_a^3}\left(\ln\left(\frac{T - T_a}{T + T_a}\right) - \ln\left(\frac{T_i - T_a}{T_i + T_a}\right) - 2\tan^{-1}\left(\frac{T}{T_a}\right) + 2\tan^{-1}\left(\frac{T_i}{T_a}\right)\right) = \frac{-\varepsilon\sigma At}{mC_p} \quad (2)$$

Here Cp = Specific Heat at Constant Pressure, m = Mass of the Material and ε = Emissivity of the SS plate

One possibility of solving the Least square residual method is to substitute various values of ε in Eqn (2) and determine the temperature Ti at various time instants given in the problem. With these following can be calculated

$$S(\varepsilon) = \sum_{i=1}^{N}\left(T_{\exp,i} - T_{calc,i}\right)^2$$

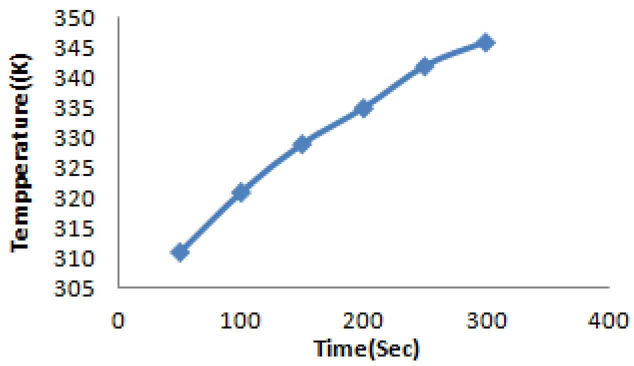

Fig .3 Plate temperature Vs Time

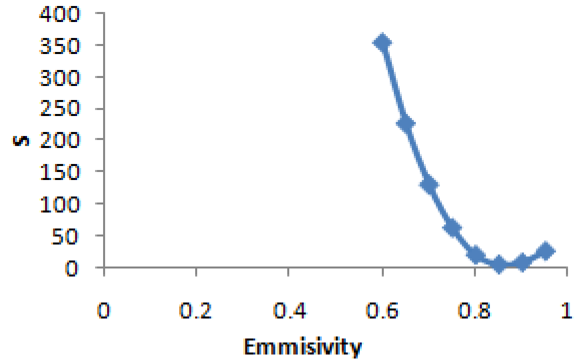

Fig.4 Variation of the Residual with emissivity

Upon doing such an exercise for ε ranging from 0.6-0.95 in the steps of 0.05.We obtain S(ε) as shown in the table. From the table and the plot we can at the best, say that 0.8<ε<0.9.The residual is plotted against the emissivity ε and is shown in Fig. 4

By locally fitting a Lagragian interpolation polynomial for S(ε), by employing three values of ε where the residuals appear to be heading towards a minimum. These happens to be 0.8, 0.85and 0.9.

$$S = \frac{(\varepsilon-0.85)(\varepsilon-0.9)}{(0.8-0.85)(0.8-0.9)}\times 19.1 + \frac{(\varepsilon-0.8)(\varepsilon-0.9)}{(0.85-0.8)(0.85-0.9)}\times 4 + \frac{(\varepsilon-0.85)(\varepsilon-0.8)}{(0.9-0.85)(0.9-0.8)}\times 7.2$$

$$S = 3660\varepsilon^2 - 6341\varepsilon + 2749.5$$

Take *ds/ dε* and equate it to zero to make S Stationary

$$\frac{ds}{d\varepsilon} = 7320\varepsilon - 6341 = 0$$

$$\varepsilon = 0.8663$$

Therefore the best estimate of ε with the level of computational intensity is 08663.

BAYESIAN APPROACH

Bayesian inference is based on Bayes conditionality probability theorem and employs probability to characterize all forms of uncertainty in the problem. The Bayes theorem to relate the experimental data Y and the parameter is

$$P\left(\frac{x}{Y}\right) = \frac{P\left(\frac{Y}{x}\right)P(x)}{P(Y)} = \frac{P\left(\frac{Y}{x}\right)P(x)}{\int P\left(\frac{Y}{x}\right)P(x)dx} \tag{3}$$

Where P(x/Y) is called the posterior probability density function (PPDF) for which the effect is Y and x is the cause, P(Y/x) is the likelihood density function(x) is the prior density function and P(Y) is the normalizing constant. In Eqn (3), the first term in the RHS represent the probability of getting Y for an assumed value of x. This can be obtained from a solution to the least square residual method problem for an assumed x and convert the

$$S = \sum_{i=1}^{N}\left(Y_{\exp,i} - Y_{sim,i}\right)^2$$

into a PDF. The P(x) is prior knowledge belief about even before the measurements are made .Bayesian inference also helps us to inject a prior if it is known already from previous knowledge and this will hasten the retrieval and also helps us in obtaining sharper PPDFs signifying lower standard deviation of the estimates.

Steps Involved in Bayesian Approach

The Bayesian method to solve an inverse problem involves three steps

- Experimental data collection(The data is collected in the form of Temperature)
- Modelling Likelihood and priori
- Estimation of x

The first step is done by conducting experiments, so far as the likelihood is concerned; we exploit the idea of measurement error in temperature as follow

$$Y_{measured} = Y_{simulated} + \omega$$

ω is a random variable from a normal distribution with mean "0" and standard deviation σ, where σ is the standard deviation of the measuring instrument. Assuming that the uncertainty ω follows a normal or Gaussian distribution, the likelihood can be modelled as

$$P\left(\frac{Y}{x}\right)=\frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n}\exp\left(\frac{\left(T-F(x)^T\right)\left(Y-F(x)\right)}{2\sigma^2}\right)$$

Where Y is a vector of dimension n i.e., n measurement are available and the F(x) is the solution to the least square residual method with the parameter vector x.

$$P\left(\frac{Y}{x}\right)=\frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n}\exp\left(\frac{\left(-\chi^2\right)}{2}\right)$$

Where N is the total number of samples in one single experiment λ^2 , Where

$$\chi^2=\sum_{i=1}^{n}\frac{\left(Y_{meas,i}-Y_{sim,i}\right)^2}{\sigma^2}$$

Where $Y_{sim,i}$ are the simulated values of Y for an assumed x.
The posterior PDF (PPDF) then becomes

$$P\left(\frac{x}{Y}\right)=\frac{\left[\frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n}\exp\left(\frac{-\chi^2}{2}\right)\right]P(x)}{\int\left[\frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n}\exp\left(\frac{-\chi^2}{2}\right)\right]P(x)dx}$$

The prior probability density(x) typically follows a uniform, normal or log normal distribution. In this case of a uniform prior, P(x) is the same for all values of x, i.e, we have absolutely no selective preference. Such a prior is called a non- informative prior.

Consider P(x) follows a normal distribution with mean μ_p and standard deviation σ_p. Mathematically P(x) is given by

$$P(x)=\frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n}\exp\left(\frac{\left(-\left(x-\mu_p\right)^2\right)}{2\sigma_p^2}\right)$$

Hence the PPDF becomes

$$P\left(\frac{x}{Y}\right)=\frac{\frac{1}{(2\Pi)^{\frac{n+1}{2}}(\sigma^n\sigma_p)}\exp(-)\left[\frac{\lambda^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\left[\int\frac{1}{(2\Pi)^{\frac{n+1}{2}}(\sigma^n\sigma_p)}\exp(-)\left[\frac{\chi^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]dx}$$

Therefore for every assumed value of the data vector $X(x_1, x_2, x_3,\ldots\ldots\ldots\ldots x_n)$,P(x/Y) can be worked out.
Therefore,

$$P\left(\frac{x}{Y}\right)=\frac{\exp(-)\left[\frac{\lambda^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\left[\int\exp(-)\left[\frac{\chi^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]dx}$$

The mean estimate of x then becomes

$$\bar{x}=\frac{\int x\exp(-)\left[\frac{\lambda^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]dx}{\left[\int\exp(-)\left[\frac{\chi^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]dx}$$

Often the integral is replaced by a summation when only discrete values of x are used,

$$\bar{x}=\frac{\sum_i x_i\exp(-)\left[\frac{\lambda^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]\Delta x_i}{\sum\left[\exp(-)\left[\frac{\lambda^2}{2}+\frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]\Delta x_i}$$

$$\bar{x} = \frac{\sum_i x_i \exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\sum\left[\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]}$$

$$\sigma_x^2 = \frac{\sum_i (x_i - \bar{x})\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\sum\left[\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]} \quad (4)$$

Here σ = Stefan-Botzmann Constant

Now we can use this framework to estimate the emissivity of Material ε.in Eqn (4) is the standard deviation of the estimated parameter, which is very diagnostic of the potency of the estimation process. Table 2 shows the Mean and Standard Deviation for single parameter using Bayesian method (With and Without Priori). Fig. 4 and Fig .5 shows the typical PPDF for Emissivity ε .Table 3 shows the comparison of optimization result with actual values.

Table-2 Mean and Standard Deviation for single parameter using Bayesian method

Parameter	Bayesian Approach	
	Without Prior	With Prior
Mean (ε)	0.85	0.85
Standard Deviation	6.9E-06	3.2573E-06

Table- 3 Comparison of Optimization Result with Actual Result

S No.	Optimization Method	ε
1	Actual Value	0.86
2	Least square residual Optimization Method	0.8663
3	Bayesian Approach	0.85

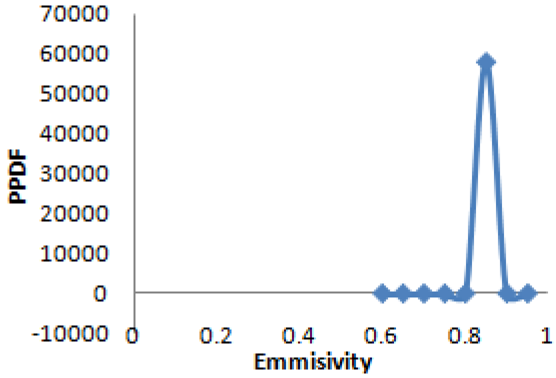

Fig.4 PPDF of Emissivity using Bayesian Method (Without Prior)

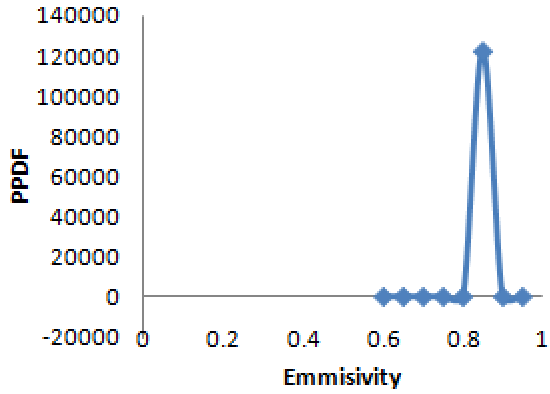

Fig.5 PPDF of Emissivity using Bayesian Method (With Prior)

CONCLUSION

Steady heat transfer laminar natural convection experiment in Vertical plate have been done for a highly polished stainless steel plate to retrieve single parameter (emissivity) by using least square residual method and Bayesian approach and compared with the value available from Actual result and found to be in good agreement.

REFERENCES

[1] W Elenbaas, Heat Dissipation of Parallel Plates by Free Convection, *Physica*, **1942**, 9, 1-27.
[2] M Fishenden and OA Saunden, *An Introduction to Heat Transfer*, Oxford Univ Press, London, **1950.**
[3] RLC Bosworth, *Heat Transfer Phenomena*, Wiley, New York, **1952.**
[4] M Mikheyev, *Fundamentals of Heat Transfer*, Peace Publishers, Moscow, **1968.**
[5] SA Nada, Natural Convection Heat Transfer in Horizontal and Vertical Closed Narrow Enclosures with Heated Rectangular Finned Base Plate, *International Journal of Heat and Mass Transfer*, **2007**,50,667-679.
[6] HF Oztop and DW Dagtekini, Natural Convection Heat Transfer by Heated Partition within Enclosure, *International Communication of Heat Mass Transfer*, **2001**, 28 (6), 823-834.
[7] AS Krishnan, C Balaji and SP Venkateshan, A Synergistic Approach to Parameter Estimation in Multimode Heat Transfer, *International Communication of Heat Mass Transfer*, **2004**, 30(4), 515-524.