



Slip Effects on MHD Flow of Casson Fluid over an Exponentially Stretching Sheet in Presence of Thermal Radiation, Heat Source/Sink and Chemical Reaction

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ABSTRACT

The present paper is devoted to describing the boundary layer flow of a non-Newtonian Casson fluid accompanied by heat and mass transfer towards a porous exponentially stretching sheet with velocity slip and thermal slip conditions in presence of thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction effects. The governing partial differential equations are reduced to a set of highly non-linear ordinary differential equations by using suitable similarity transformations and solved numerically by an implicit finite difference scheme known as the Keller box method. In the present work the effects of the non-dimensional governing parameters on velocity, temperature and concentration profiles have been discussed and presented graphically. As well as for the local skin-friction coefficient, Nusselt number and Sherwood numbers exhibited and examined. It is found that the temperature and concentration profiles are increasing to higher value when the Casson parameter increases but reverse is true for the velocity distribution. Finally, the velocity and temperature profiles are decreasing with the increasing values of the velocity slip and thermal slip parameters respectively.

Key words: MHD, exponentially stretching, casson fluid, Radiation, heat source/sink

INTRODUCTION

The study of laminar flow and heat transfer of a viscous fluid over a stretching sheet is an essential research field in fluid mechanics, due to its extensive applications in many manufacturing processes in industry, such as glass-fiber production, extraction of polymer sheet, hot rolling, wire drawing, solidification of liquid crystals, paper production, drawing of plastic films, petroleum production, exotic lubricants and suspension solutions, continuous cooling and fibers spinning. A lot of work on the boundary layer Newtonian fluids has been carried out both experimentally and theoretically. Crane [1] was the first who investigate the stretching problem taking into account the fluid flow over a linearly stretched surface. On the other hand, Gupta [2] stressed that realistically, stretching surface is not necessarily continuous. Magyari and Keller [3] analyzed the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Elbashedy [4] investigated the heat transfer over an exponentially stretching continuous surface with suction. Partha [5] discussed the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Al-Odat et al. [6] studied the effects of magnetic field on fluid flow and heat transfer over an exponentially stretching surface. Sajid and Hayat [7] studied the analytical solution of the thermal radiation effects on the flow over an exponentially stretching sheet by using the homotopy analysis method. Later, Bidin and Nazar [8] numerically studied the effect of thermal radiation on the steady laminar boundary layer flow and heat transfer over an exponentially stretching sheet. Bararnia et al. [9] analytically studied the boundary layer flow and heat transfer on a continuously stretching surface. On the other hand, El-Aziz [10] analyzed the effect of viscous dissipation on mixed convection flow of micropolar fluid past an exponentially stretching sheet. Ishak [11] discussed the combined effects of magnetic field and thermal radiation on boundary layer flow and heat transfer over an exponentially stretching sheet.

All the above investigations [1–11] deal with the laminar boundary layer flow and heat transfer over a stretching surface for a Newtonian fluid. A vast majority of reactions, involved specifically in food processing, polymer processing, biochemical industries, etc., are also typical examples of non-Newtonian behaviour. The studies of

non-Newtonian fluids offer interesting challenges to mathematicians, engineers, physicists, and computer scientists. Because of the complexity of non-Newtonian fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. In the process, there is a non-Newtonian fluid known as Casson fluid. Casson fluid exhibits yield stress. It is well known that a Casson fluid is a shear thinning liquid, which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid whereas, if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluids are as follows: jelly, human blood, honey, soup, tomato sauce, concentrated fruit juices, etc. The laminar boundary layer flow of a Casson fluid over a stretching surface attracts the attention of modern-day researchers. Dash et al [12] investigated the Casson fluid in a tube filled with a homogeneous porous medium. Eldabe and Salwa [13] have analyzed the Casson fluid for the flow between two rotating cylinders. Mukhopadhyay et al. [14] analyzed the numerical solutions for the boundary layer flow and heat transfer for a Casson fluid over an unsteady stretching surface. Pramanik [15] studied the Steady boundary layer flow of a Casson fluid and heat transfer over an exponentially stretching surface in the presence of thermal radiation.

All the above investigations assume the conventional no slip boundary conditions over a stretching surface. Undoubtedly, for many decades, scientists have conducted extensive research trying to understand and control the slip flow behaviours over a stretching surface. Partial velocity slip readily occurs for an array of complex fluid such as emulsions, suspensions, foams and polymer solutions. Also, the fluids that exhibit boundary slip have important technological applications, such as in the polishing of artificial heart valves and internal cavities. In light of these various applications many authors have investigated and reported the results on the boundary layer flow and heat transfer characteristics in the presence of slip effects. Several researchers like Ariel et al [16], Hayat et al [17], Mukhopadhyay [18] and Turkyilmazoglu [19], etc. investigated the flow problems taking slip flow condition at the boundary. Later, Mukhopadhyay [20] investigated the velocity slip and thermal slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing in presence of thermal radiation. Recently, Ene and Marinka [21] analyzed the same problem by using optimal homotopy asymptotic method. Megahed [22] investigated the effects of second order velocity slip and thermal slip on viscous boundary layer flow for Casson fluid and heat transfer past a permeable stretching sheet in presence of thermal radiation.

The aim of the present work is to investigate the numerical solution of the steady boundary layer flow for a MHD Casson fluid over a an exponentially stretching sheet with thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction involving boundary conditions of velocity slip and thermal slip effects . The governing partial differential equations are first transformed into ordinary differential equations, before being solved numerically using the Keller-box method for some values of the governing parameters.

MATHEMATICAL FORMULATION

Consider a steady two-dimensional laminar flow of an incompressible viscous and electrically conducting fluid past a exponentially stretching sheet which coincides with the plane $y = 0$. The fluid flow is confined to $y > 0$. The x-axis is taken along the continuous stretching surface in the direction of motion while the y-axis is perpendicular to the surface. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The flow is assumed to be generated by stretching of the elastic boundary sheet from a slit with a large force such that the velocity of the boundary sheet is an exponential order of the flow directional coordinate x. Along with this we considered heat source and chemical reaction to the flow. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as follows:

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i, j) th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and P_y is the yield stress of the fluid. The flow takes place in the Upper half plane $y > 0$. A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant.

The continuity, momentum and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{\mu}{\rho C_p} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho C_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} - k_1 (C - C_\infty) \quad (4)$$

Where u and v are the velocities in the x - and y directions, respectively, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density (assumed constant), μ is the coefficient of fluid viscosity, $\beta = \mu_B \sqrt{2\pi_c}/P_y$ is parameter of the Casson fluid, σ is the electrical conductivity, k is the thermal conductivity, q_r is the radiative heat flux, C_p is the specific heat at constant pressure, $Q = Q_0 e^{\frac{x}{L}}$ is the is the dimensional heat generation ($Q > 0$) or absorption ($Q < 0$) coefficient, Q_0 is a constant, D_B is the coefficient of the mass diffusivity and $k_1 = k_0 e^{\frac{x}{L}}$ is the exponential reaction rate; $k_1 > 0$ stands for destructive reaction whereas $k_1 < 0$ stands for constructive reaction, k_0 is a constant.

In writing Eq. (2), we have neglected the induced magnetic field since the magnetic Reynolds number for the flow is assumed to be very small.

Using Rosseland approximation for radiation [23] we can write

$$q_r = -\frac{4\delta^* \partial T^4}{3k^* \partial y} \quad (5)$$

Where δ^* is the Stefan-Boltzman constant, k^* is the absorption coefficient. Assuming that T^4 is a linear function of temperature, then

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \quad (6)$$

Using Eq. (5) and (6), Eq. (3) reduces to:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\delta^* T_\infty^3}{3\rho C_p k^*} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho C_p} (T - T_\infty) \quad (7)$$

Boundary Conditions

The appropriate boundary conditions for the problem are given by

$$u = U + Nv \left(1 + \frac{1}{\beta}\right) \frac{\partial u}{\partial y}, \quad v = -V(x), \quad T = T_w + D \frac{\partial T}{\partial y}, \quad C = C_w \quad \text{at } y = 0 \quad (8)$$

$$u \rightarrow 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \quad (9)$$

Where $U = U_0 e^{\frac{x}{L}}$ is the stretching velocity, U_0 is the reference velocity, $T_w = T_\infty - T_0 e^{\frac{x}{2L}}$ is the temperature at the sheet, T_0 is the reference temperature, $C_w = C_\infty - C_0 e^{\frac{x}{2L}}$ is the concentration at the sheet, C_0 is the reference concentration, $N = N_0 e^{-\frac{x}{2L}}$ is the velocity slip factor which changes with x , N_0 is the initial value of velocity slip factor and $D = D_0 e^{-\frac{x}{2L}}$ is the thermal slip factor which also changes with x , D_0 is the initial value of thermal slip factor. The no-slip case is recovered for $N = 0 = D$. $V(x) > 0$ is the velocity of suction and $V(x) < 0$ is the velocity of blowing, $V(x) = V_0 e^{\frac{x}{2L}}$, a special type of velocity at the wall is considered, V_0 is the initial strength of suction.

Method of Solution

Introducing the similarity variables as

$$\eta = \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y, \quad u = U_0 e^{\frac{x}{L}} f'(\eta), \quad v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} (f(\eta) + \eta f'(\eta)), \quad (10)$$

$$T = T_\infty + T_0 e^{\frac{x}{2L}} \theta(\eta), \quad C = C_\infty + C_0 e^{\frac{x}{2L}} \phi(\eta)$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration and primes denote differentiation with respect to η . The transformed ordinary differential equations are:

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - 2f'^2 - M^2 f' = 0 \quad (11)$$

$$\left(1 + \frac{4}{3}R\right) \theta'' + Pr(f\theta' - f'\theta) + PrEc \left(1 + \frac{1}{\beta}\right) f''^2 + Q_H \theta = 0 \quad (12)$$

$$\phi'' + Sc(f\phi' - f'\phi) - 2ScK_1 \phi = 0 \quad (13)$$

and the boundary conditions take the following form:

$$f(0) = S, \quad f'(0) = 1 + \lambda \left(1 + \frac{1}{\beta}\right) f''(0), \quad \theta(0) = 1 + \delta \theta'(0), \quad \phi(0) = 1 \quad (14)$$

$$f'(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty. \quad (15)$$

where the prime denotes differentiation with respect to η , $M = \sqrt{\frac{2\sigma B_0^2 L}{\rho U_0}}$ is the magnetic parameter, $\lambda = N_0 \sqrt{\frac{U_0 \nu}{2L}}$ is the velocity slip parameter, $S = \frac{V_0}{\sqrt{\frac{U_0 \nu}{2L}}} > 0$ (or < 0) is the suction (or blowing) parameter and $\delta = D_0 \sqrt{\frac{U_0}{2\nu L}}$ is the

thermal slip parameter, $R = \frac{4\delta^* T_\infty^3}{kk^*}$ is the radiation parameter, $Pr = \frac{\mu c_p}{k}$ is the Prandtl number, $Ec = \frac{Ue\sqrt{2L}}{c_p T_0}$ is the Eckert number, $Q_H = \frac{2LQ_0}{\rho c_p U_0}$ is the heat source/sink parameter, $Sc = \frac{\nu}{D_B}$ is the Schmidt number and $K_I = \frac{Lk_0}{U_0}$ is the chemical reaction parameter. The important physical quantities of this problem are the skin friction coefficient C_{fx} , the local Nusselt number Nu_x and the local Sherwood number Sh_x , which represent the wall shear stress, the heat transfer rate and the mass transfer rate at the surface, respectively.

The skin friction coefficient C_{fx} is given by

$$C_{fx} = \sqrt{\frac{2x}{LRe_x}} \left(1 + \frac{1}{\beta}\right) f''(0), \quad (16)$$

the local Nusselt number Nu_x is given by

$$Nu_x = -\sqrt{\frac{xRe_x}{2L}} \left(1 + \frac{4}{3}R\right) \theta'(0), \quad (17)$$

and the local Sherwood number Sh_x is given by

$$Sh_x = -\sqrt{\frac{Re_x}{2xL}} \phi'(0) \quad (18)$$

Here $Re_x = \frac{Ux}{\nu}$ is a local Reynold number.

RESULTS AND DISCUSSION

The system of ordinary differential Equations (11)- (13) along with boundary conditions (12) and (13) has been solved numerically using the Keller box method described in the book by Cebeci and Bradshaw [24]. In order to analyze the theoretical concept of the physical model, numerical computations are carried out for several sets of values of the physical parameters, namely magnetic parameter (M), Casson parameter (β), velocity slip parameter (λ), suction (/blowing) parameter (S), radiation parameter (R), thermal slip parameter (δ), Prandtl number (Pr), Eckert number (Ec), heat source/sink parameter (Q_H), Schmidt number (Sc) and reaction rate parameter (K_I). Comparison of the existing results with some available results of Magyari and Keller [3], Bidin and Nazar [8], Ishak [11] and Mukhopadhyay [20] (for some special cases) in absence of Casson fluid, magnetic field, thermal radiation, viscous dissipation, heat source/sink, chemical reaction, velocity slip, thermal slip and suction/blowing at the boundary, as presented in Table 1. The results are found in excellent agreement.

Table-1 Values of $[-\theta'(0)]$ for several values of Prandtl number Pr and radiation R in the absence of Casson fluid and chemical reaction with $M = 0, S = 0, \lambda = 0, \delta = 0, Ec = 0$ and $Q_H = 0$.

| Pr | R | Magyari and Keller [3] | Bidin and Nazar [8] | Ishak [11] | Mukhopadhyay [20] | Present study |
|----|-----|------------------------|---------------------|------------|-------------------|---------------|
| 1 | 0 | 0.9548 | 0.9547 | 0.9548 | 0.9547 | 0.9548 |
| 2 | 0 | 1.4714 | 1.4714 | 1.4715 | 1.4714 | 1.4715 |
| 3 | 0 | 1.8691 | 1.8691 | 1.8691 | 1.8691 | 1.8691 |
| 5 | 0 | 2.5001 | | 2.5001 | 2.5001 | 2.5001 |
| 10 | 0 | 3.6604 | | 3.6604 | 3.6603 | 3.6605 |
| 1 | 0.5 | | 0.6765 | | | 0.6775 |
| 1 | 1 | | 0.5315 | 0.5312 | 0.5311 | 0.5353 |
| 2 | 0.5 | | 1.0735 | | 1.0734 | 1.0735 |
| 2 | 1 | | 0.8627 | | 0.8626 | 0.8629 |
| 3 | 0.5 | | 1.3807 | | 1.3807 | 1.3807 |
| 3 | 1 | | 1.1214 | | 1.1213 | 1.1214 |

Let us now pay attention to the effects of Casson parameter β on velocity, temperature and concentration profiles. Fig.1 shows the velocity profile against the similarity variable η for various values of Casson parameter β . We observe from this figure that the boundary layer thickness increases as β decreases. Likewise, this figure depicts that for increasing values of the Casson parameter, it reduces the fluid velocity distribution inside the boundary layer

away from the sheet but the reverse is true along the sheet. Physically, with an increase in the non-Newtonian Casson parameter, the fluid yield stress is decreasing causes a production for resistance force which make the fluid velocity decreases. The temperature profile for variable values of the Casson parameter for the exponential stretching sheet is presented in Fig.2. This figure reveals that an increase in the temperature distribution along the thermal boundary layer is observed with a large enhancement in the Casson fluid parameter. Likewise, the thermal boundary layer thickness increases with increasing the Casson parameter. The nature of concentration profiles for variable values of the Casson parameter for the exponential stretching sheet is presented in Fig.3. From this figure we can observe that the concentration increases with the increases in the values of β . Likewise, the solute boundary layer thickness increases with increasing β .

The dimensionless velocity profiles for selected values of magnetic parameter M are plotted in Fig.4. It is apparent that the velocity decreases along the surface with an increase in the magnetic parameter. The transverse magnetic field opposes the motion of the fluid and the rate of transport is considerably reduced. This is because with the increase in M , Lorentz force increases and it produces more resistance to the flow. Also, it is found that the temperature distribution along the boundary layer, thermal boundary thickness and the temperature for the surface of the sheet increases with an increase in the same parameter, as we can see from Fig.5. So the temperature inside the thermal boundary layer increases due to excess of heating. Therefore the magnetic field can be used to control the flow characteristics. Fig.6 depicts the effects of the velocity slip parameter λ on the velocity profile. Velocity distribution along the boundary layer is found to decrease with increasing λ . Physically, when slip occurs, the slipping fluid shows a decrease in the surface skin-friction between the fluid and the stretching sheet because not all the pulling force of the stretching sheet can be transmitted to the fluid. So, increasing the value of λ will decrease the flow velocity in the region of the boundary layer.

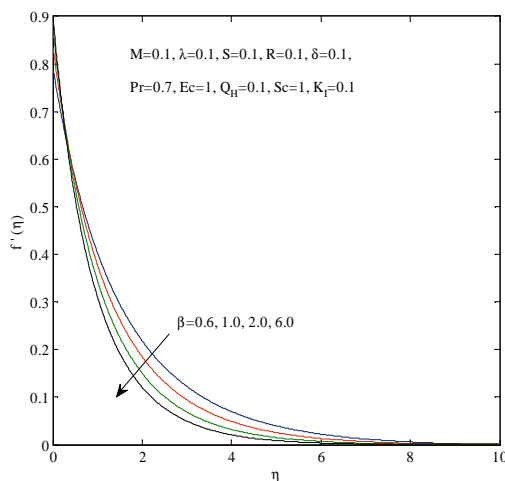


Fig.1 Velocity profiles for different values of Casson parameter β

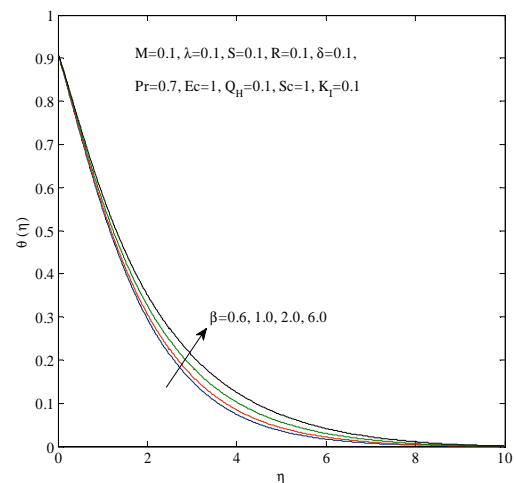


Fig.2 Temperature profiles for different values of Casson parameter β

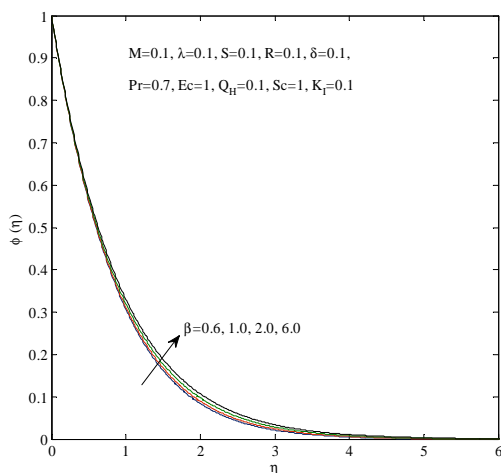


Fig.3 Concentration profiles for different values of Casson parameter β

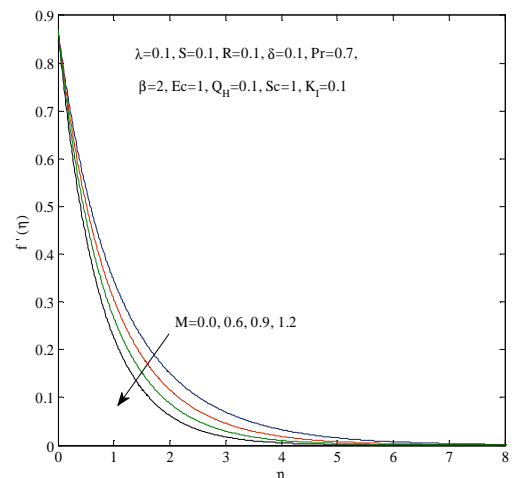


Fig.4 Velocity profiles for different values of magnetic parameter M

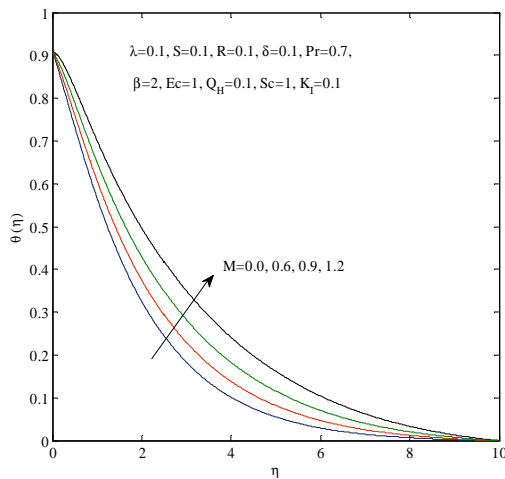


Fig.5 Temperature profiles for different values of magnetic parameter M

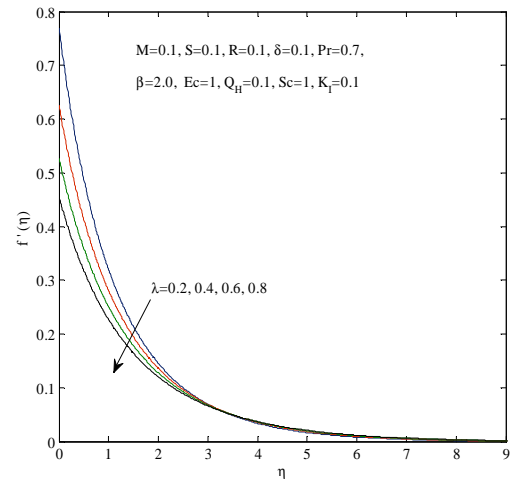


Fig.6 Velocity profiles for different values of velocity slip parameter λ

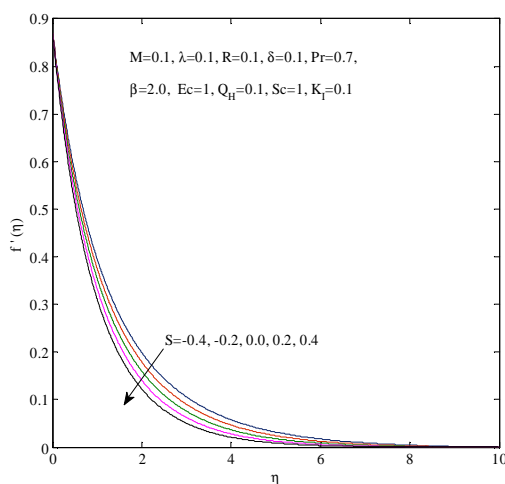


Fig.7 Velocity profiles for different values of suction/blowing parameter S

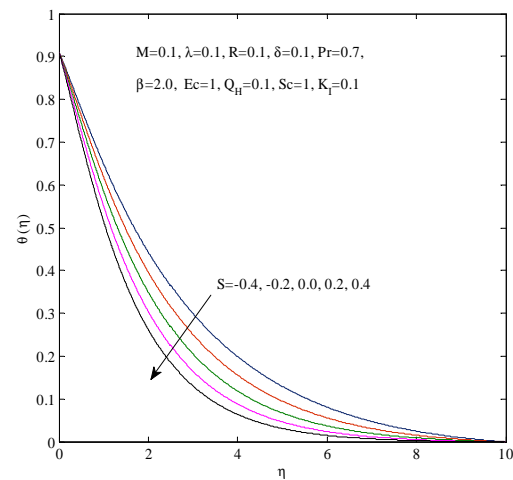


Fig.8 Temperature profiles for different values of suction/blowing parameter S

The effects of the suction (blowing) parameter on the velocity profile, the temperature distribution and the concentration profiles have been analyzed and the results are presented in Figs.7, 8 and 9. These figures show that the suction (blowing) has a profound effect on the boundary layer thickness in which the suction reduces the thermal boundary layer thickness whereas blowing thickens it. However, the net effect for the suction parameter is to slow down the flow velocity, temperature distribution and concentration but the reverse is true for the blowing parameter. So, we can conclude that the suction can be effectively used for the fast cooling of the sheet.

The effect of the thermal slip parameter δ on heat transfer may be analyzed from Fig.10. From this figure it is anticipated that the increase of thermal slip parameter δ results in the decrease in both the temperature distribution and the thermal boundary layer thickness; also, the maximum effect is observed at the surface of the stretching sheet. Fig.11 is obtained by plotting the temperature distributions against the variable η for different values of the thermal radiation parameter. From this graph, it is clear that the surface temperature $\theta(0)$, the thermal boundary layer thickness and the temperature distribution increases with an increase in the value of the thermal radiation parameter. This is because the divergence of the radiative heat flux $\frac{\partial q_r}{\partial y}$ increases as the Rosseland radiative absorptivity k^* decreases (see expression for R) which, in turn, shows an increase in the rate of radiative heat transfer to the fluid, which causes the fluid temperature to increase. In view of this fact, the effect of radiation becomes more significant as $R \rightarrow \infty$ and the radiation effect can be neglected when $R = 0$.

The effect of the Eckert number Ec on heat transfer is shown in Fig.12. It is clear that the temperature in the boundary layer region, the thermal boundary layer thickness increases with an increase in the viscous dissipation parameter. Fig.13 shows the influence of the heat source/sink parameter Q_H on the temperature profile within the thermal boundary layer. From this graph, it is observed that the temperature increases with an increase in the heat source/sink parameter. Figs.14 and 15 depicts chemical species concentration profiles against the variable η for

various values of the reaction rate parameter K_I and the Schmidt number (Sc) in the boundary layer. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. From these figures (Figs.14 and 15), it is noteworthy that the concentration boundary layer thickness decreases with an increase in chemical reaction parameter and Schmidt number.

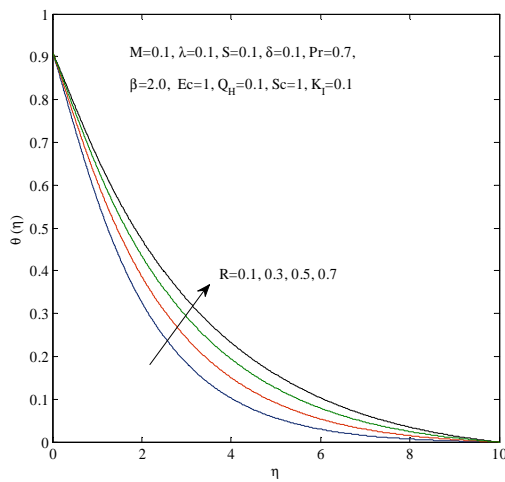


Fig.11 Temperature profiles for different values of thermal radiation parameter R

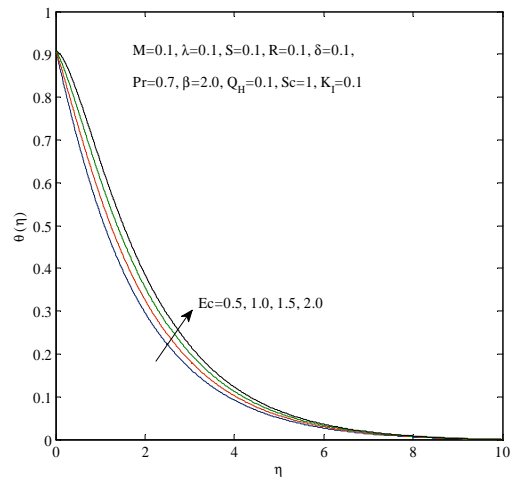


Fig.12 Temperature profiles for different values of viscous dissipation parameter Ec

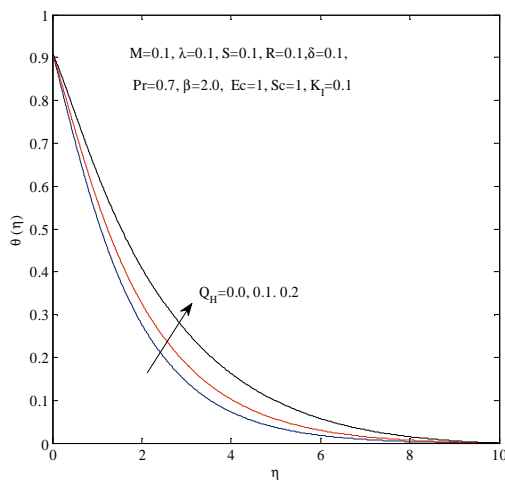


Fig.13 Temperature profiles for different values of heat source/sink parameter Q_H

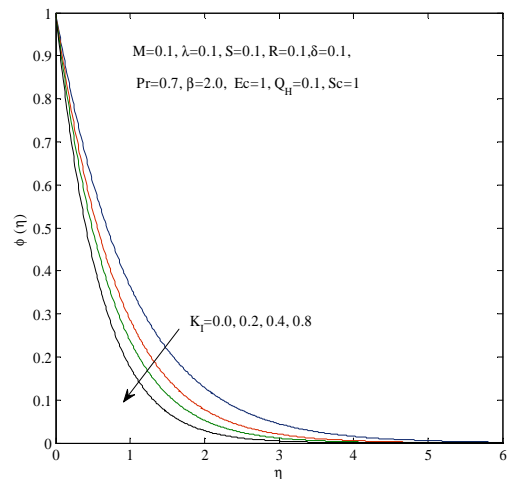


Fig.14 Concentration profiles for different values of chemical reaction rate parameter K_I

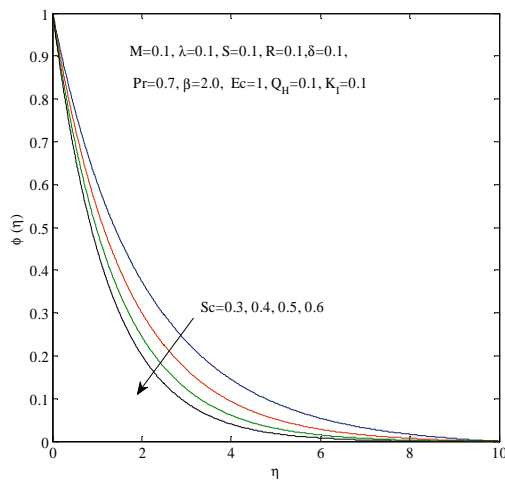


Fig.15 Concentration profiles for different values of Schmidt number Sc

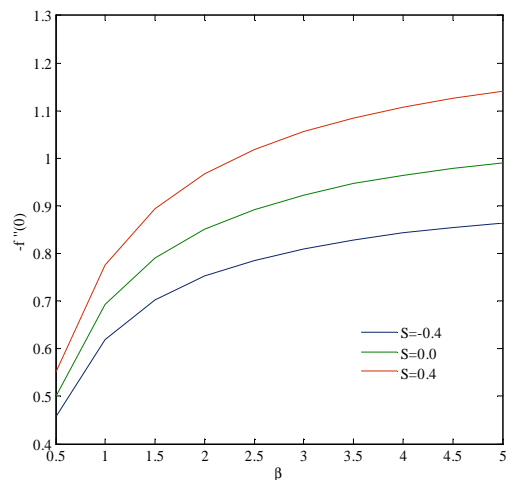


Fig.16 Skin-friction coefficient $f''(0)$ against Casson parameter β for three values of suction/blowing parameter S

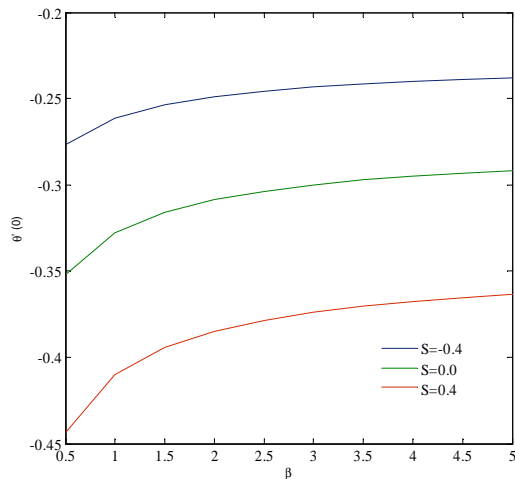


Fig.17 Wall temperature gradient $\theta'(0)$ against Casson parameter β for three values of suction/blowing parameter S

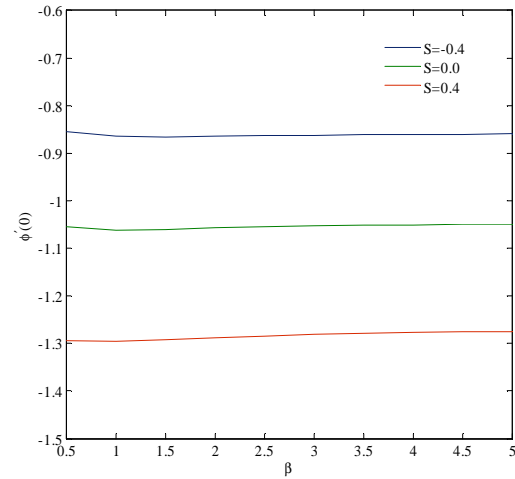


Fig.18 Wall concentration temperature gradient $\phi'(0)$ against Casson parameter β for three values of suction/blowing parameter S

Fig.16 exhibits the nature of $f''(0)$ related to skin-friction coefficient with Casson parameter β for three values of suction/ blowing parameter S . It is found that $[-f''(0)]$ increases with β and that it is higher for suction than that of blowing. From this figure, it is very clear that shear stress at the wall is negative here. Physically, negative sign of $f''(0)$ implies that surface exerts a dragging force on the fluid and positive sign implies the opposite. Fig.17 and Fig.18 displays the nature of heat transfer coefficient $[\theta'(0)]$ and mass transfer coefficient $[\phi'(0)]$ against the Casson parameter β respectively. The increase in Casson parameter β leads to increase the heat transfer coefficient and mass transfer coefficient respectively. Wall temperature gradient $[\theta'(0)]$ and wall concentration gradient are increases with blowing but decreases with suction.

CONCLUSIONS

The MHD boundary layer flow and heat transfer of a Casson fluid over an exponentially stretching sheet with slip effects, thermal radiation, magnetic field, viscous dissipation, heat source/sink and chemical reaction is analyzed here. The main findings of the present study can be summarized as follows:

- Momentum boundary layer thickness decreases with increasing Casson parameter but the thermal boundary layer thickness and the solute boundary layer thickness increases in this case.
- Magnetic parameter reduces the rate of transport but Surface shear stress increases as the magnetic parameter increases. Likewise, Wall temperature increases with increasing magnetic parameter.
- The effect of increasing values of the suction parameter is to slow down the flow velocity, temperature distribution and concentration but the reverse is true for the injection parameter.
- Due to increasing velocity slip, velocity decreases. With the increase in thermal slip parameter, temperature distribution decreases.
- The surface temperature of a sheet increases with radiation parameter R . This phenomenon is ascribed to a higher effective thermal diffusivity.
- An increase in viscous dissipation parameter and heat source parameter enhances the thermal boundary layer thickness and heat transfer rate respectively.
- The concentration boundary layer thickness decreases with an increase in Schmidt number and chemical reaction parameter.

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