



## Analysis of High Resolution Spectral Methods to Assess Power Quality

Celia Shahnaz<sup>1</sup>, Md. Saleh Ebn Sharif<sup>1</sup>, Md. Ziaul Hoque<sup>1,2</sup>, Md. Shofiqul Islam<sup>2</sup>

<sup>1</sup>Department of Electrical and Electronic Engineering  
Bangladesh University of Engineering and Technology, Bangladesh

<sup>2</sup>Department of Electrical and Electronic Engineering, International Islamic University Chittagong, Bangladesh  
[saleh.ebn.sharif@gmail.com](mailto:saleh.ebn.sharif@gmail.com), [ziaulieee@yahoo.com](mailto:ziaulieee@yahoo.com)

### ABSTRACT

*In this Paper, high resolution methods, such as Multiple Signal Classification (MUSIC), Root MUSIC and Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) are discussed that model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system. The comparison of the amplitude, power (squared amplitude) and frequency estimation error, based on Matlab simulations is carried out in order to analyse the performance of MUSIC, Root MUSIC, ESPRIT methods in estimating amplitude, power (squared amplitude) and frequency estimation of synthetic power signal both in clean and noisy conditions. Simulation results are presented on signal using mean square error (MSE) as the evaluation criterion. The variation of amplitude, power (squared amplitude) and frequency estimation are compared with respect to data sequence length and Signal to noise ratio (SNR) and their influences on MSE that demonstrate the performance of the high resolution methods are provided. Also the influences of different estimation methods on MSE are compared.*

**Key words:** High Resolution Methods, MUSIC, Root MUSIC, ESPRIT, Frequency Domain Analysis, Parameter Estimation, Parametric Methods, Power Quality, Power Signal.

### INTRODUCTION

The quality of voltage waveforms is an important issue for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The proliferation of nonlinear loads connected to power systems has triggered a growing concern with power quality issues. The inherent operation characteristics of these loads deteriorate the quality of the delivered energy, and increase the energy losses as well as decrease the reliability of a power system [1]. The power quality assessment in power systems are based on various spectrum estimation methods and the accuracy of all spectrum estimation methods is not the same [2-3].

Fourier transform is one of these methods that suffer much in fact of resolution. In the normal method like fast Fourier Transform (FFT) have some limitations. First limitation is that of frequency resolution, i.e. the ability to distinguish the spectrum responses of two or more signals [4]. Second limitation of such method is due to coherent signal sampling of the data which proves itself as a leakage in spectrum domain. We can reduce this leakage by windowing but it introduces additional distortions. These two performance limitations of FFT or similar methods are particularly troublesome when analysing short data records. To overcome from these problems High resolution spectrum estimation methods can be used where resolution problem is not found. MUSIC, Root MUSIC and ESPRIT are termed as high resolution spectrum estimation methods [1], [3-5]. These high resolution methods can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems and in particular the estimation of the power quality indices [6].

The paper is composed as follows: After the short description of parametric methods (MUSIC, Root MUSIC and ESPRIT) [8] and selected criterion measurements the comparison of the frequency amplitude & power estimation error based on Matlab simulation is presented. Next part presents basics of selected power quality indices that followed by comparison of estimation error in the case of different application of FFT based algorithms.

## POWER QUALITY ASSESSMENT

In power system frequency, amplitude and phasor are the most important parameters for monitoring, control and protection. All of these can reflect the whole power system situation. In electrical power system it is of utmost important to keep the frequency as close to its original value as possible. In order to control the power system frequency, it needs to be measured quickly and accurately. Normally power system voltage and current waveforms are distorted by harmonic and inter harmonic components, particularly during system disturbance. Faults or other switching transients may change the magnitude and phase angles of the waveforms. However, voltage and current can also be distorted by non-linear loads, power electronic components and inherent non-linear nature of the system elements [3]. Not only that the assessment of power quality can be done either by calculating, measuring or estimating power quality indices (frequency, spectrum, harmonic distortion etc.). So that the estimation of power quality indices is still an important and yet challenging part in power system. Only the estimation techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices [4]. More reliable methods are required for power quality monitoring and estimation.

The parametric high-resolution methods result from ingenious exploitations of known data structures. Example: MUSIC method, ESPRIT method etc. Parametric methods can yield higher resolutions than nonparametric methods in cases when the signal length is short. These methods use a different approach to spectral estimation, instead of trying to estimate the PSD directly from the data; they model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system [1]. These high resolution methods can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems and in particular the estimation of the power quality indices [5-6].

### Conventional Methods

Most of the conventional methods of power quality assessment in power systems are almost exclusively based on Fourier Transform. These methods suffer from various inherent limitations. These are following:

- Dependency of estimation error on the data length (phase dependence of the estimation error).
- The high degree dependence of the resolution on signal-to-noise ratio.
- The high degree dependency of the resolution on the initial phase of the harmonic components
- The improvement of the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices is difficult [4].

In order to overcome these problems, High resolution spectrum estimation method can be used where resolution problem is not found.

### High Resolution Methods

High-resolution methods are generally defined to be high-performance methods for estimating and detecting the desired and undesired signal components present in a given set of data. The term 'high-resolution' conjointly implies a decent ability to resolve terribly 'Similar' signal elements. One of the most common issues in signal processing is known as frequency estimation. In frequency estimation 'high-resolution' typically refers to a decent ability to resolve two or additional closely set frequencies within the given data [7]. On the other hand, in amplitude estimation 'high-resolution' typically refers to a decent ability to resolve two or additional closely set amplitudes within the given data. There are two types of high-resolution methods. One is parametric methods and the other is non-parametric methods.

### Non-Parametric Methods

The non-parametric high-resolution methods maximize the output of some desired information with little knowledge of the data structure. The choice between parametric methods and non-parametric methods largely depends on one's confidence in the assumed data model.

### Parametric Methods

The parametric high-resolution methods result from ingenious exploitations of known data structures. Example: MUSIC method, ESPRIT method etc. Parametric methods can yield higher resolutions than nonparametric methods in cases when the signal length is short. These methods use a different approach to spectral estimation, instead of trying to estimate the PSD directly from the data they model the data as the output of a linear system driven by white noise and then attempt to estimate the parameters of that linear system [8]. The ESPRIT and the Root-Music spectrum estimation methods are based on the linear algebraic concepts of sub-spaces and so have been called 'sub-space methods' [3], the model of the signal in this case is a sum of sinusoids in the background of noise of a known covariance function.

## MOTIVATION OF USING HIGH RESOLUTION METHODS

Accuracy of all spectrum estimation methods is not the same. Fourier transform is one of these methods that suffer much in fact of resolution. In the normal method like Fast Fourier Transform (FFT) has some limitations. First limitation of this method is that of frequency resolution, i.e. the ability to distinguish the spectrum responses of two or more signals. Second limitation of such method is due to coherent signal sampling of the data which proves itself as a leakage in spectrum domain. We can reduce this leakage by windowing but it introduces additional distortions. These two performance limitations of FFT or similar methods are particularly troublesome when analyzing short data records. To overcome from these problems High resolution spectrum estimation method can be used where no resolution problem is found. MUSIC, ESPRIT, and root MUSIC are termed as high resolution spectrum estimation methods. These high resolution methods can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems and in particular the estimation of the power quality indices [4].

In this paper our purpose is to compare the performance of the MUSIC, ESPRIT and ROOT MUSIC algorithms by performing simulations with MATLAB as a tool. The performance of these algorithms is evaluated in terms of Mean Square Error (MSE) as a function of data length and SNR.

## PROPOSED METHODS

There are various kinds of methods that are used in power amplitude and frequency estimation to improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems in particular the estimation of the power quality indices but the [9] conventional high resolution methods are more effective The description of the methods of high resolution especially MUSIC, ESPRIT and ROOT MUSIC are provided.

### Music

MUSIC (Multiple Signal Classification) is method which presents signal sub space and using this method we can get more accurate result than normal methods. Let a signal to describe the MUSIC method [13-14].

$$x = \sum_{i=1}^p A_i s_i + \eta, A_i = |A_i| e^{j\phi_i} \quad (1)$$

Where  $S_i = [1 e^{j\omega_1} \dots e^{j(N-1)\omega_2}]^T$ ,  $A_i$ -amplitudes of the signal components,  $N$ -number of signal samples,  $p$ -number of components,  $\eta$ -noise,  $\phi_i$ -components frequency. Now let a signal samples to estimate autocorrelation matrix as:

$$R_x = \sum_{i=1}^p E\{A_i A_i^*\} S_i S_i^T + \sigma_0^2 I \quad (2)$$

$N-p$  smallest eigenvalues of the correlation matrix (matrix dimension  $N > p+1$ ) correspond [5] to the noise subspace and  $p$  largest (all greater than  $\sigma_0^2$ - noise variance) correspond to the signal sub space [4].

The matrix of noise eigenvectors of the above matrix (2) is used

$$E_{\text{noise}} = [e_{p+1} e_{p+2} \dots e_N] \quad (3)$$

to compute the projection matrix for the noise subspace:

$$P_{\text{noise}} = E_{\text{noise}} E_{\text{noise}}^{*T} \quad (4)$$

Which by using an auxiliary vector  $W = [1 e^{j\omega t} \dots e^{j(N-1)\omega t}]^T$  allows computation of projection of vector  $w$  onto the noise subspace as:

$$w^{*T} P_{\text{noise}} w = w^{*T} E_{\text{noise}} E_{\text{noise}}^{*T} w = \sum_{i=p+1}^N E_i (e^{j\omega}) E_i^* (e^{j\omega}) \xrightarrow{z} \sum_{i=p+1}^N E_i(z) E_i^*(1/z^*) \quad (5)$$

The last polynomial in (5) has  $p$  double roots lying on the unit circle which angular positions correspond to the frequencies of the signal components. This method of finding the frequencies is therefore called root-MUSIC.

After the calculation of the frequencies the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix using the relations [10]

$$e_i^{*T} R_x e_i = \lambda_i \text{ And } R_x = \sum_{i=1}^p P_i S_i S_i^{*T} + \sigma_0^2 I \quad (6)$$

and solving for  $p_i$  – components power.

### Root-Music

Root-MUSIC method facilitates the same ideas with MUSIC and differs only in the second step of the MUSIC algorithm. The main advantage of Root-MUSIC over MUSIC is its lower computational complexity. The MUSIC spectrum is an all pole function of the form [11].

$$P_{\text{mu}}(\theta) = \frac{1}{\text{abs}[F(\theta)^H U_N U_N^H F(\theta)]} \quad (7)$$

Let  $C = U_N U_N^H$  using equation (7) written as

$$P_{\text{mu}}^{-1} = \sum_{m=1}^M \sum_{n=1}^M \exp\left(\frac{j(m-1)2\pi d \sin(\theta_b)}{\lambda}\right) C_{mn} A \quad (8)$$

Where  $A = \exp\left(\frac{-j(m-1)2\pi d \sin(\theta_b)}{\lambda}\right)$  and  $C_{mn}$  is the entry in the  $m^{\text{th}}$  row and  $n^{\text{th}}$  column of  $C$ . Combination of two sums into one gives equation (8):

$$P_{\text{mu}}^{-1} = \sum_{n=1}^M C_1 \exp\left(\frac{-j2\pi n d \sin(\theta_b)}{\lambda}\right) \quad (9)$$

Where  $C_1 = \sum_{m=-n=1} C_{mn}$  is the sum of the entries of C. Along the  $l^{\text{th}}$  diagonal polynomial representation D (z) will

$$D(z) = \sum_{l=-M+1}^{M+1} C_1 z^{-1} z \quad (10)$$

If the eigen decomposition corresponds to the true spectral matrix, then MUSIC spectrum becomes equivalent to the polynomial D(z) on the unit circle and peaks in the MUSIC spectrum exists as ROOTs of the D(z) lie close to the unit circle. A pole of D(z) at  $z=z_1 = |z_1| \exp(j \arg(z_1))$  will result in a peak in the MUSIC spectrum at  $\theta = \sin^{-1}(\{\lambda/2rd\} \arg[z_1])$ .

### ESPRIT Method

The original ESPRIT (Estimation of Signal Parameter via Rotational Invariance Technique) was developed by another one as in example. It is based on a naturally existing shift invariance between the discrete time series which leads to rotational invariance between the corresponding signal subspaces. The shift invariance is illustrated below. After the Eigen-decomposition of the autocorrelation matrix as:

$$R_x = U^* T A \quad (11)$$

It is possible to partition a matrix by using special selector matrices which select the first and the last (M-1) columns of a (M × M) matrix respectively:

$$\begin{aligned} \Gamma_1 &= [I_{M-1} | 0_{(M-1) \times 1}]_{(M-1) \times M} \\ \Gamma_2 &= [0_{(M-1) \times 1} | I_{M-1}]_{(M-1) \times M} \end{aligned} \quad (12)$$

By using of matrices  $\Gamma$  two subspaces are defined spanned by two subsets of eigenvectors as follows:

$$\begin{aligned} S_1 &= \Gamma_1 U \\ S_2 &= \Gamma_2 U \end{aligned} \quad (13)$$

For the matrices defined as  $S_1$  and  $S_2$  in (13) for every  $\omega_k, K \in \mathbb{N}$  representing different frequency components and matrix  $\phi$  defined as:

$$\Phi = \begin{bmatrix} e^{j\omega_1} & 0 & \dots & 0 \\ 0 & e^{j\omega_2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & e^{j\omega_k} \end{bmatrix} \quad (14)$$

The following relation can be proven [14]:

$$[\Gamma_1 U] \phi = \Gamma_2 U \quad (15)$$

The matrix  $\phi$  contains all information about frequency components. In order to extract this information, it is necessary to solve (10) for  $\phi$ . By using a unitary matrix (denoted as T) the following equations can be derived:

$$\begin{aligned} \Gamma_1 (UT) \phi &= \Gamma_2 (UT) \\ \Gamma_1 U (T \phi T^{*T}) &= \Gamma_2 \end{aligned} \quad (16)$$

In the further considerations the only interesting subspace is the signal subspace spanned by signal eigenvectors  $U_s$ . Usually it is assumed that these eigenvectors correspond to the largest Eigen values of the correlation matrix and  $U_s = [u_1 u_2 \dots \dots \dots u_k]$ . ESPRIT algorithm determines the frequencies  $e^{j\omega_k}$  as the eigenvalues of the matrix  $\phi$ . In theory the equation (15) is satisfied exactly. In practice matrices  $S_1$  and  $S_2$  are derived from an estimated correlation matrix so this equation does not hold exactly it means that (11) represents an over-determined set of linear equations [12].

## RESULTS AND DISCUSSIONS

### Performance Evaluation Criterion

The criterion used for evaluating the MUSIC ESPRIT and Root MUSIC is mean square error (MSE) with respect to data length and SNR. The equation of MSE is given by-

$$\text{For amplitude estimation, } A_{\text{MSE}} = \sqrt{\left[\left(\frac{1}{N}\right) \sum_{i=1}^N (A_{\text{iest}} - A_{\text{org}})^2\right]}$$

$$\text{For Frequency estimation } F_{\text{MSE}} = \sqrt{\left[\left(\frac{1}{N}\right) \sum_{i=1}^N (f_{\text{iest}} - f_{\text{org}})^2\right]} \text{ and}$$

$$\text{For power estimation } P_{\text{MSE}} = \sqrt{\left[\left(\frac{1}{N}\right) \sum_{i=1}^N (A_{\text{iest}}^2 - A_{\text{org}}^2)\right]} \text{ Where } A_{\text{iest}} = \text{Estimated amplitude, } A_{\text{org}} = \text{Original amplitude, } f_{\text{iest}} = \text{Estimated frequency, } f_{\text{org}} = \text{Original frequency.}$$

### Data Length

Parametric high resolution methods (MUSIC, ESPRIT and Root MUSIC) methods use a different approach to spectral estimation; instead of trying to estimate the frequency, amplitude and power directly from the data, they model the data as the output of a linear system driven by white noise, and then attempt to estimate the parameters of that linear system. In addition, the parametric high resolution methods (MUSIC, ESPRIT and Root MUSIC) lead to a system of linear equations which is relatively simple to solve. MUSIC, ESPRIT and Root MUSIC methods yield higher resolutions than nonparametric methods in cases when the signal length is short.

## SNR

Signal-to-noise ratio (often abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of a desired signal to the level of background noise. It is defined as the ratio of signal power to the noise power. It is the power ratio between a signal and noise. A ratio higher than 1:1 indicates more signal than noise. While SNR is commonly quoted for electrical signals it can be applied to any form of signal (such as isotope levels in an ice core or biochemical signaling between cells) [13]. It can be derived from the formula

$$\text{SNR} = P_{\text{Signal}}/P_{\text{Noise}} = \mu/\sigma \quad (17)$$

Where  $\mu$  = the signal mean or expected value,  $\sigma$  = the standard deviation of the noise.

## Amplitude Method

Amplitude is the most important parameter in power system monitoring control and protection. It can reflect the whole power system situation. In electrical power system it is of utmost importance to keep the amplitude as close to its nominal value as possible. In order to control the power system amplitude, it needs to be measured quickly and accurately. For this the estimation of amplitude is still an important and yet challenging part [14].

## Power Estimation

In general power system voltage and current waveforms are distorted by harmonic and inter harmonic components particularly during system disturbance. Faults or other switching transients may change the magnitude and phase angles of the waveforms. However, voltage and current can also be distorted by non-linear loads power electronic components and inherent non-linear nature of the system elements [3]. Not only that the assessment of power quality can be done either by calculating measuring or estimating power quality indices (frequency spectrum harmonic distortion etc.). Only the estimation techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems in particular the estimation of the power quality indices [4].

## Frequency Estimation

Frequency is the most important parameter in power system monitoring control and protection. It can reflect the whole power system situation. In electrical power system it is of utmost importance to keep the frequency as close to its nominal value as possible. In order to control the power system frequency, it needs to be measured quickly and accurately. But in general power system voltage and current waveforms are distorted by harmonic and inter harmonic components particularly during system disturbance. Faults or other switching transients may change the magnitude and phase angles of the waveforms [8]. So the estimation of frequency is still an important and yet challenging part in power system. The effect of data length and SNR on MSE for frequency estimation is described on later sections.

## Data Length Effects for Amplitude Power and Frequency Estimations

Parametric high resolution methods (MUSIC, ESPRIT and Root MUSIC) methods use different approach to spectral estimation, instead of trying to estimate the frequency amplitude and power directly from the data they model the data as the output of a linear system driven by white noise and then attempt to estimate the parameters of that linear system. In addition, the parametric high resolution methods (MUSIC, ESPRIT and Root MUSIC) lead to a system of linear equations which is relatively simple to solve. MUSIC, ESPRIT and Root MUSIC methods yield higher resolutions than nonparametric methods in cases when the signal length is short [4]. The data sequence length influences the mean square error and therefore the accuracy of high resolution methods depends on data length. The performance of the high resolution methods (MUSIC, Root MUSIC and ESPRIT) could be identified by comparing the mean square error [8] of frequency estimation both for shorter and higher data length. Both for clean and noisy signal the performance of the mean square error of frequency estimation of the high resolution methods (MUSIC, Root MUSIC and ESPRIT) are shown. When roughly summarizing different results from the Fig. 1,2 and 3, a list of data of Amplitude Power and Frequency estimation for clean signal in terms of MSE for increasing data length can be represented as show that in Table -1, 2 and 3.

**Table -1 The Amplitude Estimation for Clean Signal in Terms of MSE for Increasing Data Length**

Data Length	MUSIC Method	Root MUSIC Method
50	10.17	-0.04498
100	8.989	-0.08449
150	7.142	-0.1277
200	6.557	-0.1753
250	6.183	-0.2282
300	6.742	-0.2876
350	5.973	-0.3549
400	4.307	-0.431

**Table -2 The Power Estimation for Noisy Signal in Terms of MSE for Increasing Data Length**

Data Length	MUSIC Method	Root MUSIC Method
50	20.39	-0.02034
100	18.13	-0.02924
150	14.41	-0.04442
200	13.23	-0.06647
250	12.43	-0.09625
300	13.63	-0.135
350	12.07	-0.1844
400	8.673	-0.2464

Table -3 The Frequency Estimation for Clean Signal in terms of MSE for Increasing Data Length

Data Length	MUSIC	Root MUSIC	ESPRIT
50	1.897	1.898	1.898
100	1.896	1.898	1.898
150	1.895	1.898	1.898
200	1.891	1.898	1.898
250	1.891	1.898	1.898
300	1.886	1.898	1.898
350	1.885	1.898	1.898
400	1.883	1.898	1.898

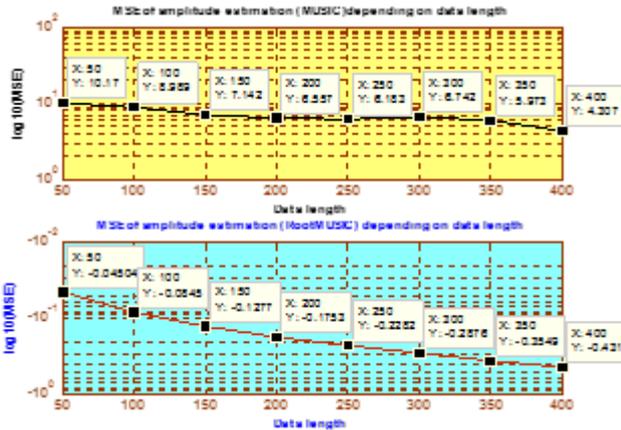


Fig. 1(a) Amplitude estimation (MUSIC & Root MUSIC) for clean signal in terms of MSE with respect to data length for clean signal in log scale

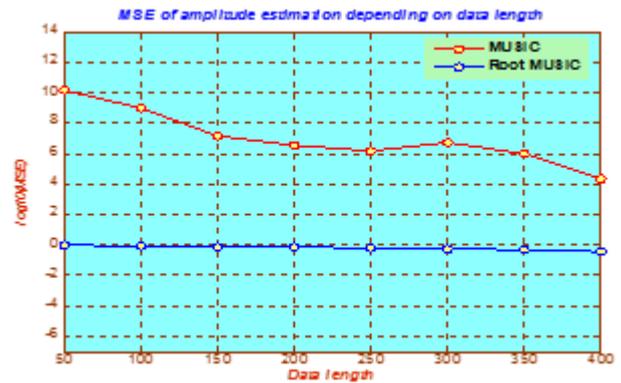


Fig. 1(b) Amplitude estimation (MUSIC & Root MUSIC) for Noisy signal in terms of MSE with respect to data length in linear scale

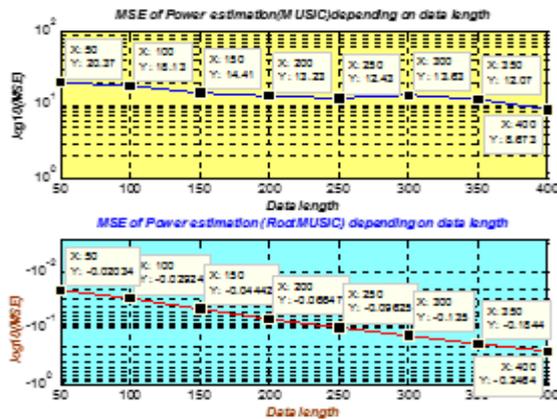


Fig. 2(a) Power estimation (MUSIC & Root MUSIC) for clean signal in terms of MSE with respect to data length in log scale

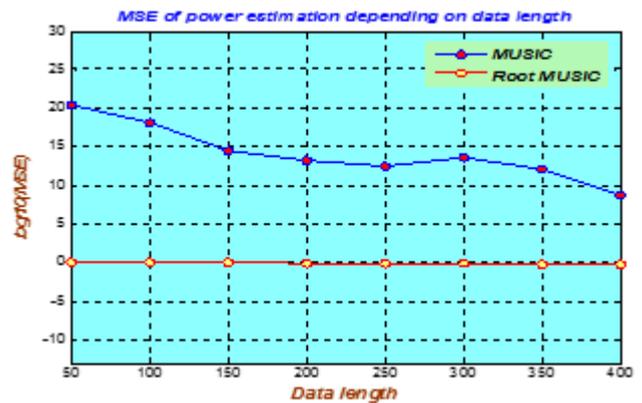


Fig. 2(b) Power estimation (MUSIC & Root MUSIC) for Noisy signal in terms of MSE with respect to data length in linear scale

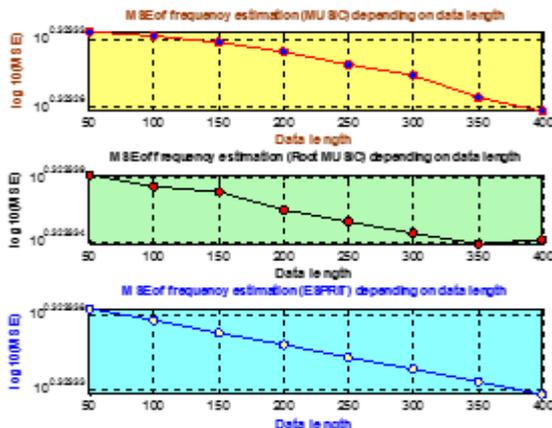


Fig. 3 (a) Frequency estimation (MUSIC , Root MUSIC & ESPRIT) for clean signal in terms of MSE with respect to data length in log scale

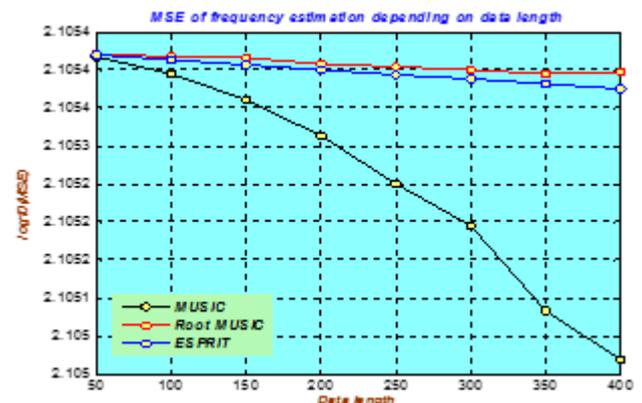


Fig. 3 (b) Frequency estimation (MUSIC , Root MUSIC & ESPRIT) for Noisy signal in terms of MSE with respect to data length in linear scale

The data sequence length influences the mean square error and therefore the accuracy of high resolution methods depends on data length. The performance of the high resolution methods (Root MUSIC, MUSIC and ESPRIT) could be identified by comparing the mean square error of amplitude estimation both for shorter and higher data length. Both for clean and noisy signal the performance of the mean square error of amplitude estimation of the high resolution methods (Root MUSIC, MUSIC and ESPRIT) are shown in Table -1 & 2.

In Fig. 1(a-b) & 2(a-b) and Table -1 & 2 show that there is a sharp decrease of the estimation error for increasing length of the data sequence (pattern for MUSIC and Root MUSIC method results are similar). Root MUSIC method performs better for power estimation in terms of MSE. In Fig. 3(a&b) and Table -3 shows that there is a sharp decrease of the estimation error for increasing length of the data sequence (pattern for MUSIC Root MUSIC and ESPRIT method results are similar). Though it is seen that MUSIC method performs better for frequency estimation but for many simplifications different assumptions and the complexity of the problem ESPRIT method is better than MUSIC and Root MUSIC.

**SNR Effects for Amplitude Power and Frequency Estimations**

It is defined as the ratio of signal power to the noise power. It is the power ratio between a signal and noise. A ratio higher than 1:1 indicates more signal than noise. While SNR is commonly quoted for electrical signals it can be applied to any form of signal (such as isotope levels in an ice core or biochemical signalling between cells). It can be derived from the formula  $SNR = P_{Signal}/P_{Noise} = \mu/\sigma$ ; Where  $\mu$  = the signal mean or expected value,  $\sigma$  = the standard deviation of the noise. There is a strong dependency of the accuracy of the frequency estimation on SNR. The performance of the high resolution methods (MUSIC Root MUSIC and ESPRIT) could be identified by comparing the mean square error of amplitude estimation both for very low and very high noise levels. Both for low and very high noise level the performance of the mean square error of amplitude estimation of the high resolution methods (MUSIC Root MUSIC and ESPRIT) are shown.

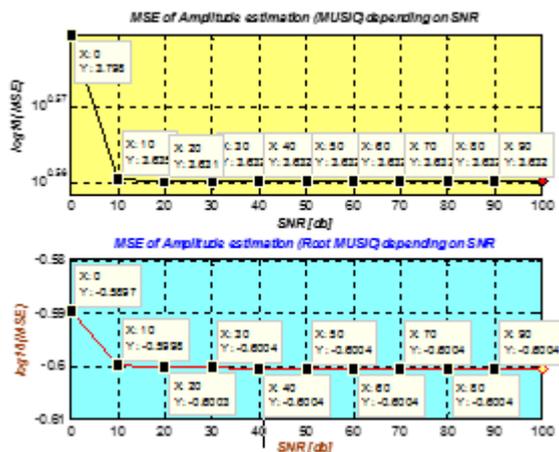


Fig. 4(a) Amplitude estimation (MUSIC & Root MUSIC) in terms of MSE with respect to SNR in log scale

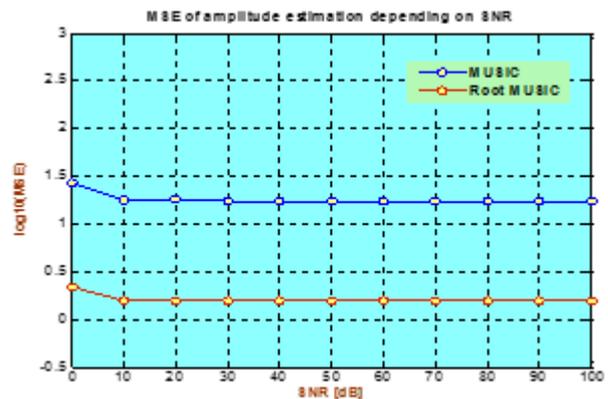


Fig. 4 (b) Amplitude estimation (MUSIC & Root MUSIC) in terms of MSE with respect to SNR in linear scale

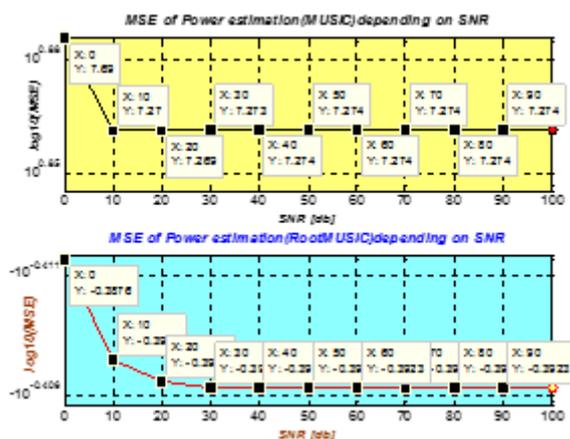


Fig. 5 (a) Power estimation (MUSIC & Root MUSIC) in terms of

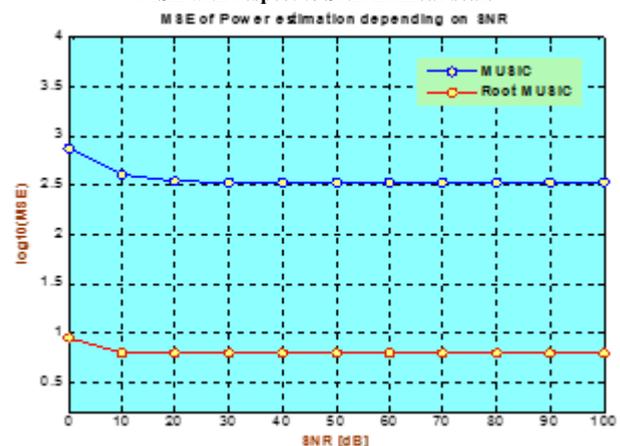


Fig. 5 (b) Power estimation (MUSIC & Root MUSIC) in terms of MSE

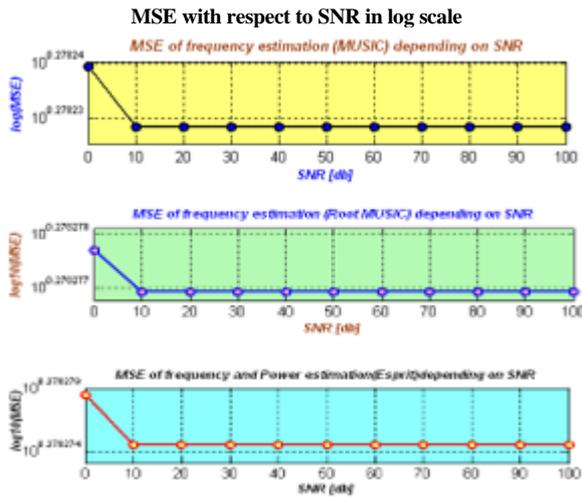


Fig. 6(a) Frequency estimation (MUSIC, Root MUSIC & ESPRIT) in terms of MSE with respect to SNR in log scale

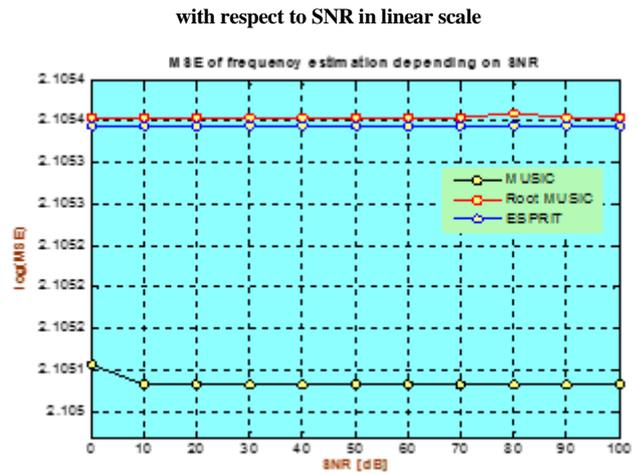


Fig. 6(b) Frequency estimation (MUSIC, Root MUSIC & ESPRIT) in terms of MSE with respect to SNR in linear scale

Table -4 Amplitude estimation (MUSIC & Root MUSIC) in terms of MSE for changing SNR

Data Length	MUSIC Method	Root MUSIC Method
0	3.798	-0.5897
10	3.635	-0.5998
20	3.631	-0.6003
30	3.632	-0.6004
40	3.632	-0.6004
50	3.632	-0.6004
60	3.632	-0.6004
70	3.632	-0.6004
80	3.632	-0.6004
90	3.632	-0.6004
100	3.632	-0.6004

Table -5 Power Estimation (MUSIC & Root MUSIC) in Terms of MSE for Changing SNR

Data Length	MUSIC Method	Root MUSIC Method
0	7.69	-0.3876
10	7.27	-0.3967
20	7.273	-0.3943
30	7.274	-0.3933
40	7.274	-0.3923
50	7.274	-0.3923
60	7.274	-0.3923
70	7.274	-0.3923
80	7.274	-0.3923
90	7.274	-0.3923
100	7.274	-0.3923

Table -6 Frequency Estimation (MUSIC, Root MUSIC and ESPRIT) in Terms of MSE for changing SNR

SNR	MUSIC	Root MUSIC	ESPRIT
0	2.1051	2.10543	2.10537
10	2.1045	2.10541	2.10537
20	2.1045	2.10541	2.10537
30	2.1045	2.10541	2.10537
40	2.1045	2.10541	2.10537
50	2.1045	2.10541	2.10537
60	2.1045	2.10541	2.10537
70	2.1045	2.10541	2.10537
80	2.1045	2.10541	2.10537
90	2.1045	2.10541	2.10537
100	2.1045	2.10541	2.10537

When roughly summarizing different results from the Fig. 4, 5 and 6 a list of data of Amplitude Power and Frequency estimation in terms of MSE for changing SNR can be represented as showed that in Table -4, 5 and 6. In Table -4 shows that there is a sharp decrease of the estimation error for increasing length of the data sequence (pattern for MUSIC and Root MUSIC method results are similar). Root MUSIC method performs better for amplitude estimation in terms of MSE in terms of MSE. Table -4 shows that there is a sharp decrease of the estimation error for increasing length of the data sequence (pattern for MUSIC and Root MUSIC method results are similar). Root MUSIC method performs better for amplitude estimation in terms of MSE in terms of MSE.

When roughly summarizing different results from the Fig. 4, 5 and 6 a list of data of Amplitude Power and Frequency estimation in terms of MSE for changing SNR can be represented as show that in Table -4, 5 and 6. In Table -5 and Table -6 shows that there is a sharp decrease of the estimation error for changing SNR (pattern for MUSIC, Root MUSIC and ESPRIT method results are similar). MUSIC and Root MUSIC method performs better for Power and Frequency estimation in terms of MSE.

## CONCLUSION

In this paper we compared the performance of high resolution spectrum estimation methods (MUSIC, Root MUSIC and ESPRIT). The performance was estimated as accuracy of estimation of frequencies, amplitudes and power of Synthetic Power Signals. The evaluation criterion and simulation results of the proposed methods are described in detail. It is observed that both methods (Root MUSIC & ESPRIT) are similar in the sense that they are both Eigen decomposition based methods which rely on decomposition of the estimated correlation matrix into two subspaces: noise and signal subspace. On the other hand, MUSIC uses the noise subspace to estimate the signal components while ESPRIT uses the signal subspace. In addition, the approach in many points is different. Root MUSIC method performs better for amplitude and power estimation in terms of MSE with respect to data length and SNR. Due to many simplifications different assumptions and the complexity of the problem simulation results represent the performance of ESPRIT method is better for frequency estimation in terms of MSE with respect to data length. Finally, the performance of MUSIC method is better for frequency estimation with respect to SNR.

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