



Numerical Study of Natural Convection on a Heated Flat Plate Subjected to Periodic Boundary Conditions

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ABSTRACT

Recently, an increased attention is turned to the problem of the natural convection heat transfer from a heated vertical plate due to its increasing importance in many industrial and technological applications. In the present paper, the numerical simulation of natural-convective heat transfer of a heated vertical plate subjected to periodic boundary conditions is carried out. The numerical approach applied here is based on solving the set of the governing equations using the control volume technique over a non-staggered grid system. In our algorithm, the implicit fractional step-non iterative method is applied to obtain the velocity and the temperature fields. The exact solutions for the classical Stoke's problem and the similarity solution are used to verify the numerical results obtained. The comparison of the numerical results obtained showed that the present numerical method applied can be used in such applications with confidence.

Key words: Natural convection, Numerical simulation, heated vertical plate, periodic boundary conditions

INTRODUCTION

Recently, an increased attention has been given to investigate the oscillation-induced heat and mass transfer transport phenomenon in different industrial and engineering applications. The importance of this phenomenon is achieved in the fields of Bioengineering, ocean engineering and chemical industrial engineering. For laminar oscillating flow in tubes, a wide range of published investigations has been carried out [1-4]. Their results showed that an enhancement of the diffusion coefficient is occurred to the flow oscillations.

In such context, it is suggested that a similar enhancement for the heat conduction could also be obtained. That has been approved by [5], and consequently, a new technology for the heat transfer enhancement by fluid oscillation has been developed.

The first paper investigates the laminar natural convection on a heated vertical plate subjected to a periodic oscillation is published by [6]. Their results showed that there is an enhancement of heat transfer by increasing the dimensionless oscillation frequency, amplitude, the Prandtl number, and a decreased with the Grashof number [6].

GOVERNING EQUATIONS

Fig. 1 shows a schematic diagram of oscillating-plate problem in stagnant air. The flat plate with length L is subjected to different oscillated boundary conditions either for velocity or temperature. The flat plate temperature is considered to be T_w and the ambient air temperature is T_∞ . It is assumed that the temperature difference is sufficiently small that the Boussinesq approximation can be applied. In the presence of the gravitational field, and according to the density difference resulting from temperature variation, the fluid starts to move.

The governing equations describe the fluid motion can be considered as:

$$\nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) = -\nabla P + (\rho + \Delta \rho) \mathbf{g} + \mu \nabla^2 \mathbf{V} \quad (2)$$

$$\frac{\partial(\rho T)}{\partial t} + \nabla \cdot (\rho \mathbf{V} T) = \rho \alpha \nabla^2 T \tag{3}$$

where, \mathbf{V} is the velocity vector, P the pressure, T the temperature, ρ the density, μ the viscosity, g the gravity and α is the thermal diffusivity. The density difference can be represented according to Boussinesq approximation by:

$$\Delta \rho = \rho(T - T_o) / T_o \tag{4}$$

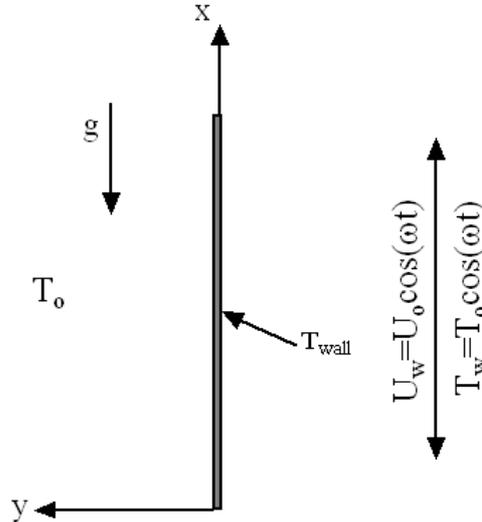


Fig.1 Oscillating Flat Plate Problem

COMPUTATIONAL METHOD

The numerical method adopted here is based on the control volume formulation for the governing equations. The so-called Implicit Fractional Step-Non Iterative method is applied for solving the governing equations and predicting the thermo-fluid dynamics of the considered problem. More details about the numerical methods can be found in [7-13].

RESULTS AND DISCUSSION

In the present section, the numerical results obtained for the problem under consideration are presented. Two different cases are studied, as shown in Table 1.

Table -1 The Boundary Conditions for the Oscillating Flat Plate Problems

Case Study	U_w	T_w	Exact solution
Case I	0.0	$T_w = \text{Const}$	Similarity solution, Ref. [14]
Case II	$U_w = U_o \cos(\omega t)$	$T_w = T_o \cos(\omega t)$	Exact solution, Ref. [15]

The different cases are considered for the validation of the numerical method adopted. The dimensionless numbers used in such simulation is the Prandtl number and the Grashof number, defined as:

$$Pr = \frac{\nu}{\alpha} \tag{5}$$

$$Gr = \frac{g \beta (T_w - T_o) L^3}{\nu^2} \tag{6}$$

$$\nu = \frac{\mu}{\rho} \tag{7}$$

Fig. 2, 3 show the velocity and the temperature distribution of buoyant convection on a fixed vertical plate for two different fluids, air ($Pr=0.72$) and water ($Pr=10$). The results showed good agreement when compared with the similarity solution of [14].

Fig. 4, 5 shows the comparison of the numerical results obtained for the heated flat plate subjected to periodic boundary conditions either for velocity or temperature. The obtained results are compared with the exact solution of the Stoke's second problem [15]. The agreement obtained is acceptable, and consequently, the present numerical method can be used for predicting the thermo-fluid characteristics of oscillating flat plate and similar problems.

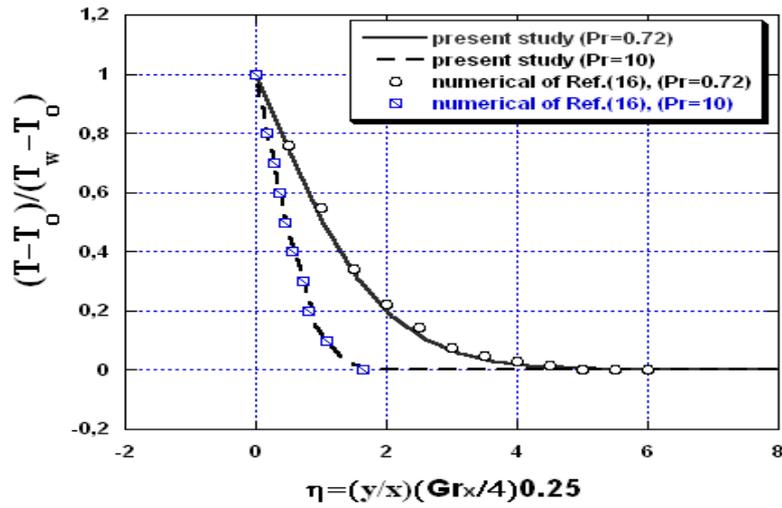


Fig.2 Comparison of the obtained dimensionless velocity with the similarity solution of Ref. [14]

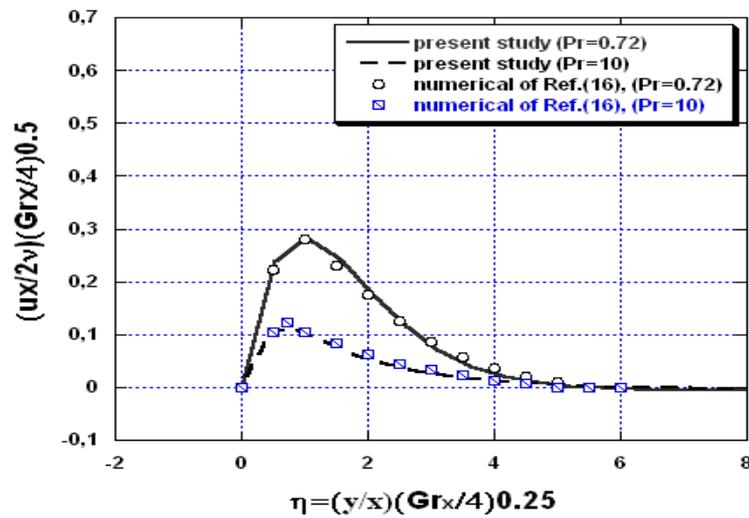


Fig.3 Comparison of the obtained dimensionless temperature with the similarity solution of Ref. [14]

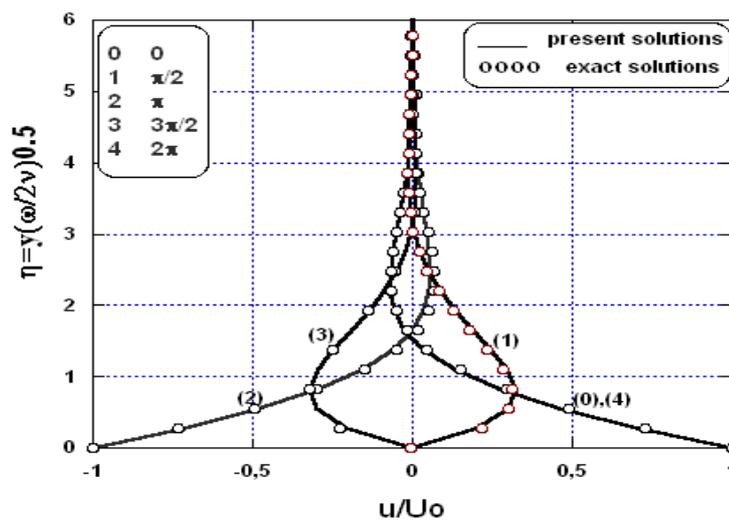


Fig. 4 Comparison of the obtained dimensionless velocity for the Stoke's second problem with the exact solution of Ref. [15]

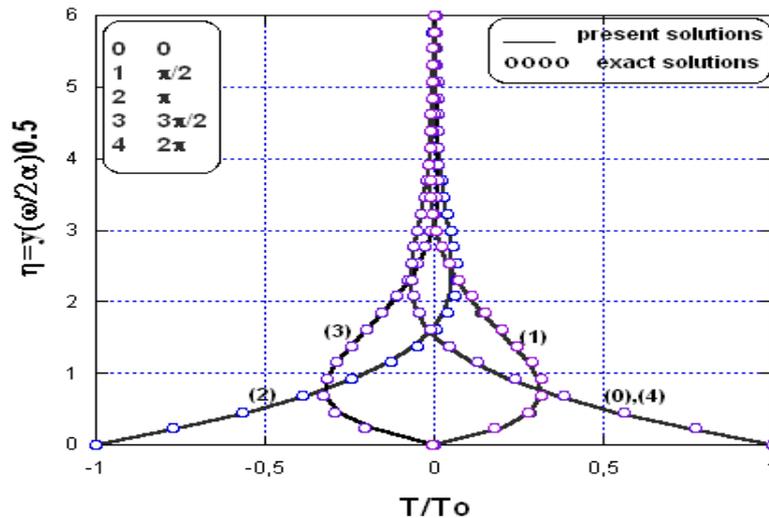


Fig. 5 Comparison of the obtained dimensionless temperature for the Stoke's second problem with the exact solution of Ref. [15]

CONCLUSION

The present paper introduces the numerical simulation of the problem of the natural convection heat transfer from a heated flat plate subjected to periodic boundary conditions. The numerical method applied here is based on solving the governing equations using the control volume approach over a non-staggered grid system. The exact solutions for the classical Stoke's problem and the similarity solution are used to verify the numerical results obtained. In the general, the results obtained revealed the capability of the numerical method adopted for predicting such complex problem and further similar problems in such context.

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