



## Active Vibration Control of Un-crack and Crack Cantilever Beam Using Piezoelectric Actuator

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### ABSTRACT

Active vibration control of fixed free rectangular cantilever beam considered and analyzed using ANSYS. Natural frequency and amplitudes of free vibration determine both from ANSYS 14.5 and experimental setup. Uncrack and crack beam with same cross section used for analysis. PZT 5H attached near the fixed end of the beam and voltage with different values applied for controlling the amplitudes of vibration. The simulation and experimental results studied for the uncrack and crack beam. The results are in the good agreement for the control of amplitude of vibration when the voltage range increases. The voltage range is from 50V to 200V at a step increase of 50V. The natural frequency of the crack beam is larger than uncrack beam. In uncrack beam, increase in crack depth at the same location natural frequencies decreases. Both the experimental and ANSYS results indicate that piezoelectric patch as an actuator is an effective method for the vibration suppression.

**Key words:** Cantilever Beam, Harmonic Response, Free Vibration, FEA

### INTRODUCTION

Today's increasing many industrial, aerospace and defence applications, vibration of flexible structures are important issue. A little excitation or fault in flexible structure leads to large amplitude vibration and long settling time. Vibration control is the use of vibration analysis to eliminate or reduce unwanted vibrations [1-2], normally in a measuring sensor or manufacturing tool. In this sense, it is possible to distinguish between active, semi active and passive strategies, depending on the presence of active elements entering energy in the system, elements just dissipating it but with actively adjustable characteristics, like magneto rheological dampers, or passive elements, respectively. In past studies of active vibration control various smart materials include electro rheological fluid (ERF), magneto rheological fluid (MRF), shape memory alloys (SMA), piezoelectric actuators, optical fibers easily subjected to parameter vibrations and disturbances [3-4].

In the recent years, the era of researches on smart materials, such as piezoelectric transducers extensively used as distributed actuator and sensor for vibration control of flexible structures [5]. The piezoelectric patches are attached to the fixed of the host structure. The optimum location of sensor/actuator on a flexible structure plays an important role on the performance of control system [4]. The increase in stiffness of structure with adding the stiffeners, natural frequency increases and modal strain energy decreases. The modal strain energy is different at different mode shapes [6].

A Donoso presented the study on controlling the tip-deflection of a cantilever plate subjected to static and time-harmonic loading on its free extreme. First, the thickness profile of a piezoelectric bimorph actuator is optimized and second, the width profile. In the thickness study, formulation and results depend on whether the electric field or the applied voltage is kept constant. Results are presented for both design variable cases, for static as well as for dynamic excitation for single frequency and frequency intervals.

Nejhad et al [8] provided a guideline for the use of piezoelectric stack and monolithic patch actuators in finite element analysis. The results from the analytical formulae compared with the manufacturer's experimental data and the obtained finite element results in this work show excellent agreements. An active vibration suppression study using FEA was employed to determine the optimum voltage (OV) applied to a piezoelectric actuator.

Wang et al [10] presented the theoretical modal analysis for the use of PVDF sensor (Polyvinylidene Fluoride) in structural modal testing via finite element analysis (FEA). A series of rectangular PVDF films are adhered on the surface of cantilever beam as sensors, while the point impact force is applied as the actuator for experimental modal analysis (EMA). Natural frequencies and mode shapes determined from both FEA and EMA are validated. In FEA, the beam structure is modeled by 3D solid elements, and the PVDF films are modeled by 3D coupled field piezoelectric elements.

Both modal analysis and harmonic response analysis are performed to obtain the structural modal parameters and frequency response functions, respectively. Results show that both FEA and EMA results agree well.

### VIBRATION ANALYSIS OF CANTILEVER BEAM

The stability and local flexibility of the beam depends on the material properties, physical dimensions, boundary conditions of the structure which play important role for the determination of its dynamic response. The characteristics of beam greatly depend on applied boundary conditions. The rectangular beam is clamped at left end and free at right end and uniform in rectangular cross section along its length. The natural frequencies and amplitude of un crack and crack beam were obtained from both experimental and ANSYS. The crack beam is analyzed experimentally and in Ansys, considering the four cases with varying depth and location.

The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity EI and density of material  $\rho$ . While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.

By using Euler's Bernoulli beam theory, 
$$\frac{\partial^2 w}{\partial t^2} + \frac{EI}{\rho} \frac{\partial^4 w}{\partial x^4} = 0 \quad (1)$$

To find the response of the system one may use the variable separation method by using the following equation.

$$w(x, t) = \phi(x)q(t) \quad (2)$$

$\phi(x)$  is known as the mode shape of the system and  $q(t)$  is known as the time modulation. Now equation (1)

$$\phi(x) \frac{\partial^2 q(t)}{\partial t^2} + \frac{EI}{\rho} \frac{\partial^4 \phi(x)}{\partial x^4} q(t)$$

reduces to

$$\text{or} \quad -\frac{EI}{\rho} \left( \frac{1}{\phi(x)} \right) \left( \frac{\partial^4 \phi(x)}{\partial x^4} \right) = \frac{1}{q} \left( \frac{\partial^2 q}{\partial t^2} \right) \quad (4)$$

Since the left side of equation (4) is independent of time t and the right side is independent of x the equality holds for all values of t and x. Hence each side must be a constant. As the right side term equals to a constant implies that

the acceleration  $\left( \frac{\partial^2 q}{\partial t^2} \right)$  is proportional to displacement  $q(t)$  one may take the proportionality constant equal to  $-\omega^2$  to have simple harmonic motion in the system. If one takes a positive constant, the response will grow exponentially and make the system unstable. Hence one may write equation (4) as,

$$-\frac{EI}{\rho} \left( \frac{1}{\phi(x)} \right) \left( \frac{\partial^4 \phi(x)}{\partial x^4} \right) = \frac{1}{q} \left( \frac{\partial^2 q}{\partial t^2} \right) = -\omega^2 \quad (5)$$

Hence,

$$\frac{d^2 q}{dt^2} + \omega^2 q = 0 \quad (6)$$

And

$$\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{\rho \omega^2}{EI} \phi(x) = 0 \quad (7)$$

Taking,

$$\beta^4 = \frac{\rho \omega^2}{EI}$$

The above equation can be written as

$$\frac{\partial^4 \phi(x)}{\partial x^4} - \beta^4 \phi(x) = 0 \quad (8)$$

The solution of equation (6) and (8) can be given by

$$q(t) = C_1 \sin \omega t + C_2 \cos \omega t \quad (9)$$

$$\phi(x) = A \sinh(\beta x) + B \cosh(\beta x) + C \sin(\beta x) + D \cos(\beta x) \quad (10)$$

$$\text{Hence, } w(x, t) = (A \sinh \beta x + B \cosh \beta x + C \sin \beta x + D \cos \beta x)(C_1 \sin \omega t + C_2 \cos \omega t) \quad (11)$$

Here constants  $C_1$  and  $C_2$  can be obtained from the initial conditions and constants  $A, B, C, D$  can be obtained from the boundary conditions. Let us now determine the mode Shape of cantilever beam.

In case of cantilever beam the boundary conditions are:

At left end i.e.

$$\begin{aligned} \text{at } x = 0 \quad w(x, t) &= 0 & (\text{displacement} = 0) \\ \frac{\partial w(x, t)}{\partial x} &= 0 & (\text{Slope} = 0) \end{aligned}$$

At the free end i.e.,

$$\begin{aligned} \text{at } x = L \quad \frac{\partial^2 w(x, t)}{\partial x^2} &= 0 & (\text{Bending Moment} = 0) \\ \frac{\partial^3 w(x, t)}{\partial x^3} &= 0 & (\text{Shear Force} = 0) \end{aligned}$$

$$\text{At } x = 0 \quad \varphi(x) = 0 \text{ and } \frac{\partial \varphi(x)}{\partial x} = 0 \quad (12)$$

$$\text{At } x = L \quad \frac{\partial^2 \varphi(x)}{\partial x^2} = 0 \text{ and } \frac{\partial^3 \varphi(x, t)}{\partial x^3} = 0 \quad (13)$$

Substituting these boundary conditions in the general solution,

$$\varphi(x) = A \cosh \beta x + B \sinh \beta x + C \cos \beta x + D \sin \beta x \quad (14)$$

From equation (12)

$$A = -C \text{ and } B = -D \quad (15)$$

$$\varphi(x) = A(\cosh \beta x - \cos \beta x) + B(\sinh \beta x - \sin \beta x) \quad (16)$$

From (13 and 16) one may have

$$\frac{A}{B} = -\frac{\sinh \beta l + \sin \beta l}{\cosh \beta l + \cos \beta l} = -\frac{\cosh \beta l + \cos \beta l}{\sinh \beta l - \sin \beta l} \quad (17)$$

or,

$$\cos \beta l \cosh \beta l = -1 \quad (18)$$

and the root of the equation is,  $\beta l = \frac{(2n-1)\pi}{2}$

Hence one may solve the frequency equation  $\cos \beta l \cosh \beta l = -1$  to obtain frequencies of different modes. For the first two modes the values of  $\beta l$  are calculated as 1.875, 4.694. For a simple elastic beam problem with uniform cross-sectional area, a well-known natural frequency can be calculated by

$$\omega = (\beta l)^2 \sqrt{EI/\rho AL^4} \quad \text{in rad/sec}^2$$

### FINITE ELEMENT MODELLING

The ANSYS 14.5 was used for uncracked and cracked beams. The dimension of the beam and piezoelectric patch is 400 x 30 x 5 mm and 76.2 x 25.4 x 0.5 mm respectively. In pre-processor, first the eight key points were created and then straight line segments were formed. These straight lines were joined sequentially to create an area. Finally, the area was extruded along the normal plane and a three dimensional uncracked and cracked beam model was obtained in ANSYS as shown in Fig.1. A three dimensional SOLID 185 and solid 45 element was selected to model the beam and piezoelectric patch respectively. Crack beam was meshed using tetrahedral element which is the best for crack configuration. Piezoelectric patch was modeled on the beam by offsetting the work plane. Both the beam and piezoelectric patches are combined using contact manager through contact analysis. Cantilever boundary conditions were modeled by constraining all degrees of freedom on the left end area. The first three natural frequencies for each case were obtained.

### EXPERIMENTAL SET UP

Experiments are performed to determine the natural frequencies and amplitude for uncracked and cracked aluminum beam (400x30x5mm). The experimental setup is shown in Figure 2. The natural frequency and amplitude of vibration for uncracked and cracked beam recorded by Lab view software. These results obtained from the software are compared with Ansys. The experimental results are obtained by varying the voltage from 50V to 200V at a step increase of 50V.

RESULTS AND DISCUSSIONS

All the results obtained from experiment and Ansys are presented this section separately and shows a good agreement. First the natural frequency of the uncrack and crack beam is obtained using modal analysis. Secondly experimental results are obtained from harmonic analysis for both uncrack and crack beam. For the crack beam, results are obtained at crack location 100 mm and 200 mm for the depth of 1 mm and 2 mm at each location, from fixed end. The change in natural frequencies and amplitudes of uncrack and crack beam at various conditions of beam as shown in Fig. 1- 3. The natural frequency of the crack beam is the larger than uncrack beam. In uncrack beam, natural frequencies decrease, in increase in crack depth at the same location.

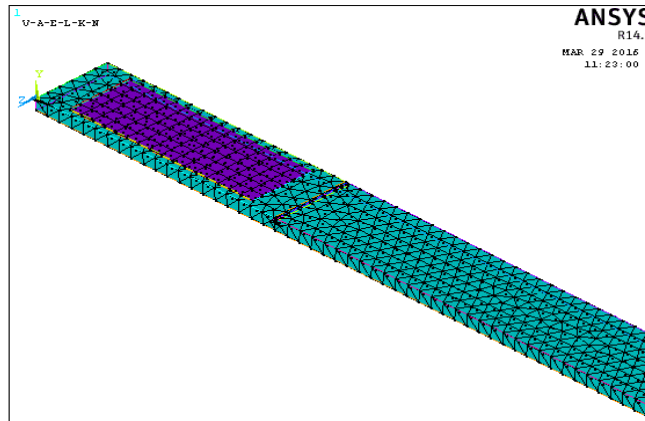


Fig. 1 Meshing of Aluminum Cantilever Crack Beam in ANSYS

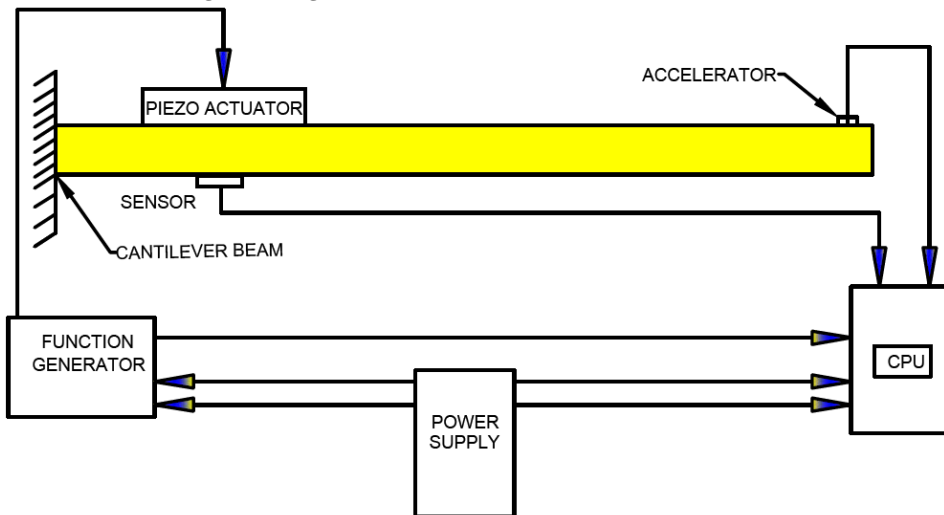


Fig. 2 Experimental Setup

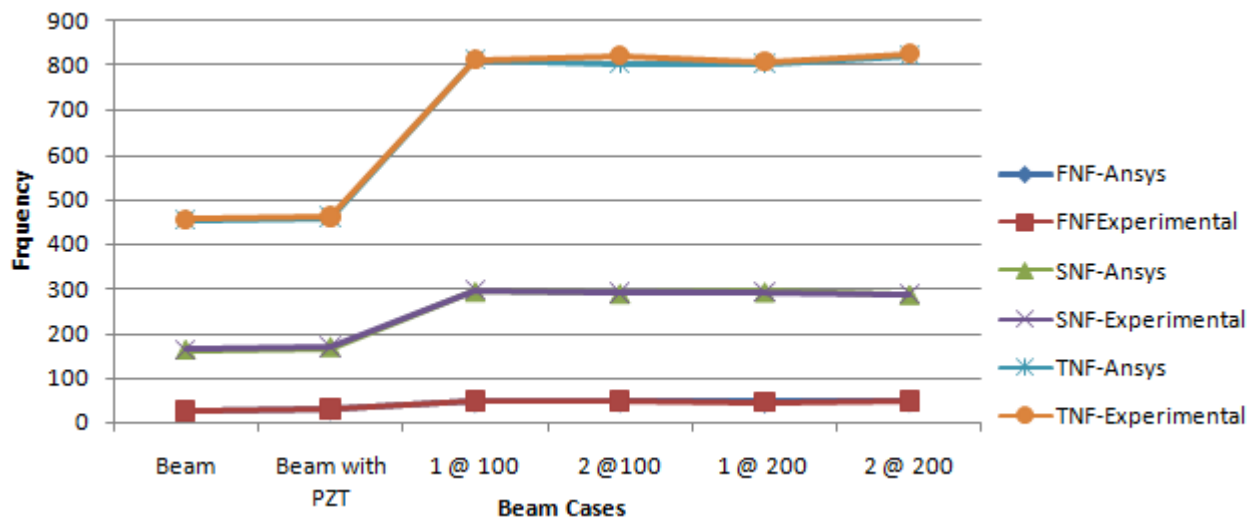


Fig. 3 Modal Analysis of Cantilever Beam

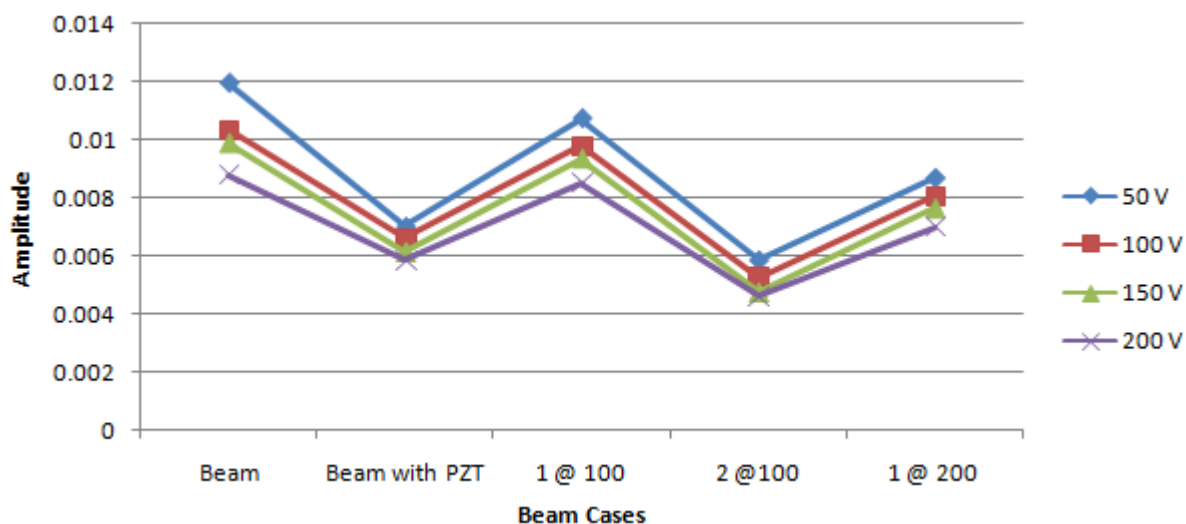


Fig. 4 Harmonic Analysis of Cantilever (uncontrolled)

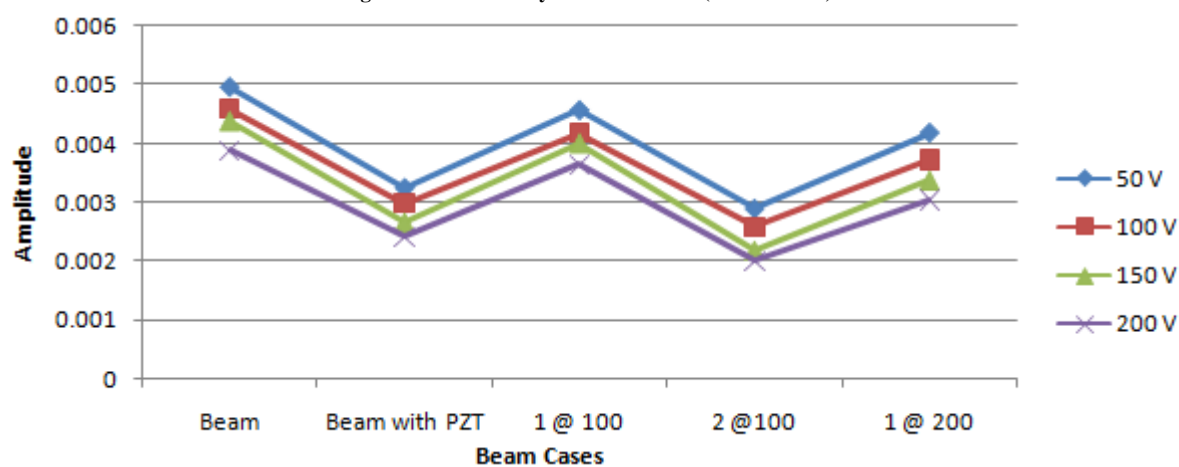


Fig. 5 Harmonic Analysis of Cantilever Beam (Controlled)

## CONCLUSIONS

In the present section, the natural frequency of the beam is increases lightly and amplitude decreases if the piezoelectric patch is attached to the beam at fixed end due to increases in stiffness. Natural frequency and amplitude decreases with increase in crack depth.

Results obtained from the different analyses can be sated as follows:

- When the crack location is constant from the fixed end and crack depth increases: the natural frequency and amplitude decreases.
- When the crack depth is constant and crack location increases from fixed end: the natural frequency and amplitude increases with increases in crack depth.

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