



## Iterative Least Square Polynomial Approximation Method for filtering ECG Signals

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### ABSTRACT

Ubiquitous healthcare system has become the new paradigm with the advancements in the field of Information and Communication Technology (ICT). Such systems involve pervasive, incessant, in vitro data collection of vital physiological parameters and real-time processing of the data to derive consequential, context-sensitive, person-specific conclusions. Electrocardiogram (ECG) is a vital parameter to be consistently and continuously monitored. ECG is often contaminated by the presence of transient interruptions and other artefacts. This paper proposes a new filtration method known as Iterative Least Square Polynomial Approximation method. It looks into the various aspects in the cleansing of ECG signals by this filter in Matlab and compares its performance with that of various commonly used filters.

**Key words:** ECG filter, Patient Monitoring, ECG processing

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### INTRODUCTION

The fact sheet [1] released by the World Health organisation (WHO) in January 2015 confirms that the reason for maximum number of non-communicable deaths is cardiovascular diseases, with a global average of 1.7 million people. Hence it is vital to monitor the ECG signals for the timely detection of anomalies.

ECG signals are captured using different methods - both intrusive and non-intrusive. Signals captured by these techniques can be contaminated due to various factors like [2] noise from the data collecting device, interference due to power line, loss of electrode contact or electrode contact noise, variation in the signals due to the movements or respiration of the subject, multi volt level potential generated due to muscle contractions, abrupt shifts in baseline and electro surgical noise. This may lead to variations in the original signal. This may shrink the performance of sophisticated signal processing algorithms. These artefacts can be reduced considerably, but not completely removed, by carefully choosing the hardware and signal collection mechanisms and then applying proper filters to the signal. Not much alternatives are on hand in hardware and signal collection mechanisms. Hence it is important to choose proper filtering techniques suitable for the artefacts as well as the intended application. The filtering can be made before sampling with a time-variant analog filter or after sampling, using a discrete-time filter or a digital filter. Digital filters [3] are more versatile and adaptive to change; they can also be used against very low frequencies. Digital filters help in the reconstruction and restoration of signals that were contaminated or damaged.

### RELATED WORKS

Many researchers have ventured [4] into the development of monitoring systems for ECG using non-invasive methods and worked with many and varied digital filters. Sameni *et al* [5] in their paper, describe about a non-linear framework based on Bayesian filters for de-noising ECG. They put forth a dynamic model, non-linear in nature, for the creation of highly realistic resultant ECG. A group of model parameters are chosen to help the framework adapt to a vast range of ECGs. The facility of auto-selection of parameters is also provided in the system.

In their research paper, Zoltán *et al* [6] elaborate on the filtering of ECG signals based on Discrete Wavelet Transformation (DWT). Using DWT, dyadic division of bandwidth is possible, thus allowing the processing of the sub bands independently. The wavelets are decomposed to obtain the sub bands. Using wavelet shrinkage and

thresholding, de-noising of ECG is done. Inverse wavelet transformation is then applied on wavelets to reconstruct the signal. Borries et al [7] also used the technique of Discrete Wavelet Transformation for filtering white noise, abrupt shifts in baseline and interference due to power line. Niknazar et al [8] describe the application of Dynamic Time Wrapping on the Kalman Filter Framework in the filtering of abnormal ECG. They use this method to filter the abnormal waves occurring only in certain cycles of ECG. The test results based on real and synthetic test data shows that this method can be used for both normal and abnormal ECG.

Ali et al [9] elaborates on the filtering of the variations in ECG arising due to the muscle contractions and interference. They use a combination of Recursive Least Square filters along with a modified version of Linear, Iterative Kalman Filter to remove the EEG traces from the surface of ECG signal. Sameni et al [10] also describe the use of the Extended Kalman Filter (EKF) in the filtering of ECG signal. While the traditional Kalman filter is valid only for linear systems, it is extended to contain nonlinear systems as well. Smital and Kozumplík [11], in their paper, illustrate how the Wiener-Shrink method could be used for filtering noise signals from ECG. The threshold levels were set and the results achieved were evaluated based on their Signal-to-Noise Ratio (SNR).

Tarvainen *et al* [12] coined a new detrending method that works like a time-variant FIR high pass filter. Detrending is done using the Smoothness Priors method. The frequency response could be adequately adjusted to suit various circumstances with the help of a single parameter. Mehmet and others [13] present an ECG de-noising technique for weak signals based on the thresholding method that is interval dependent. The algorithm assumes that the signal is contaminated by white Gaussian noise having low signal-to-noise ratio. Wavelet transformation is done by selecting proper interval-dependent thresholds.

## METHODOLOGY

Time domain and frequency domain filters are used for the proposed analysis. Wavelet transformation is not considered in the experiment due to its disadvantages when applied on a periodic signal [14]. The filters considered for analysis were Butterworth (frequency domain IIR filter), Median, fir1 (window-based FIR), MA filter (Moving Average filter), Gaussian and S-G filter. The filtered outputs of the individual filters are analysed. The output is compared with the output of the proposed method.

The proposed method, known as the Iterative Least Square Polynomial Approximation method, smoothens the data by using the least square polynomial approximation iteratively. The number of iterations is preset. Also a signal window is defined, similar to that of the moving average filters. The mathematical interpretation of the method is given below.

Let the signal,  $S = \{x[n]\}$ , be a collection of points. Consider that the collection of  $2N+1$  samples centres at  $n = 0$ . The coefficients of the polynomial can be defined as

$$P(n) = \sum_{k=0}^n a_k n^k \quad (1)$$

Approximated mean-square error for the collection of samples [15] centred at  $n = 0$  is,

$$E_n = \sum_{n=-N}^N (P(n) - x[n])^2 \quad (2)$$

ie,

$$E_n = \sum_{n=-N}^N \left( \sum_{k=0}^n a_k n^k - x[n] \right)^2 \quad (3)$$

Here,  $N$  could be treated as half width of the approximation interval. The output at  $n = 0$ ,  $y[0]$ ,

$$Y[0] = P[0] = a_0 \quad (4)$$

This is equal to the coefficient of the zeroth polynomial. Shift the analysis interval by one sample to the right; redefine the origin to position at the middle of the sample block, repeat the polynomial fitting and the output of the next sample is obtained.

In the next iteration, the output of the first iteration is considered as the new input, producing a new polynomial and new value for the output sequence. At the end of the iteration, the deviation, given by the equation (2), is minimised. This process is repeated till the threshold value is reached, after which the signal gets distorted losing out few minimas and/or maximas.

## EXPERIMENT

The data for the experiment is taken from the MIT-BIH Arrhythmia Database [16-18]. The ECG samples stored in the database are collected at Beth Israel hospital, Boston. Of the 48 record available in the database, the case of record 100 is considered for analysis. The record under consider is already filtered of unnecessary artefacts, so random noise is added to the signal before testing.

The performance of selected filters is compared using both qualitative and quantitative methods. Qualitative analysis involves visual evaluation. The parameters used for quantitative analysis are Signal to Noise ratio (SNR), Normalised RMSE (Root Mean Square Error) and Coefficient of variation (CV) of RMSE.

SNR is calculated as 
$$SNR = 10 \log_{10} \left( \frac{\text{Sum\_sqr of the signal}}{\text{Sum\_sqr of the noise}} \right) \tag{5}$$

RMSE is calculated as 
$$RMSE = \sqrt{\frac{\sum_{i=1}^n (x[i]_{act} - x[i]_{exp})^2}{n}} \tag{6}$$

Normalised RMSE is calculated as 
$$NormRMSE = \left( \frac{RMSE}{y_{max} - y_{min}} \right) \tag{7}$$

Coefficient of Variation of RMSE is calculated as 
$$CV(RMSE) = \left( \frac{RMSE}{y_{mean}} \right) \tag{8}$$

The output of various filters is given from Fig. 1-10.

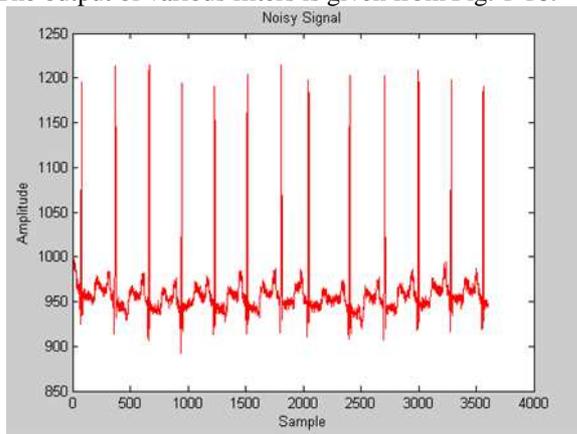


Fig. 1 Noisy Signal

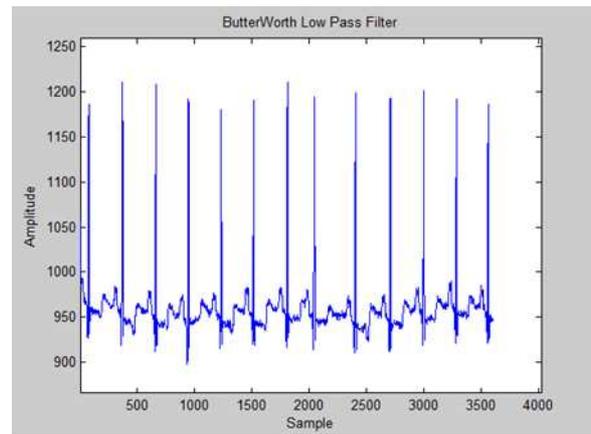


Fig. 2 Output of Butterworth Low Pass Filter

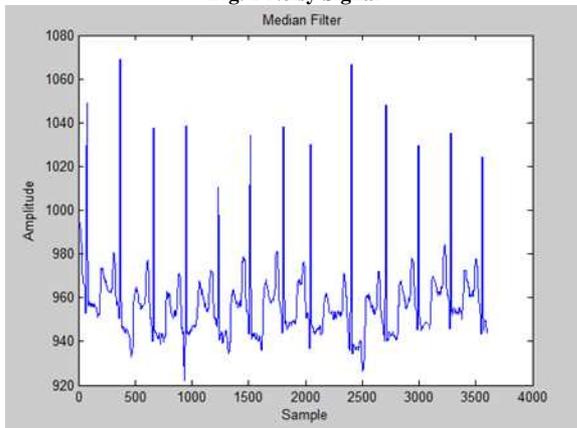


Fig. 3 Output of Median Filter

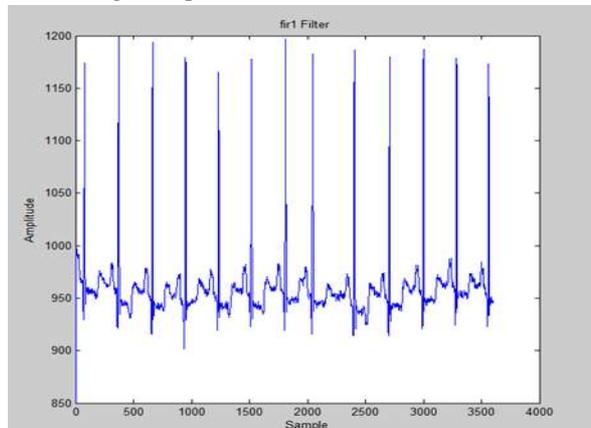


Fig. 4 Output of Fir1 Filter

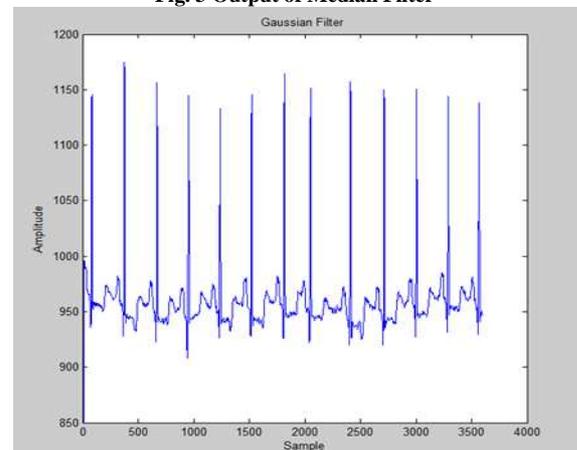


Fig. 5 Output of Gaussian Filter

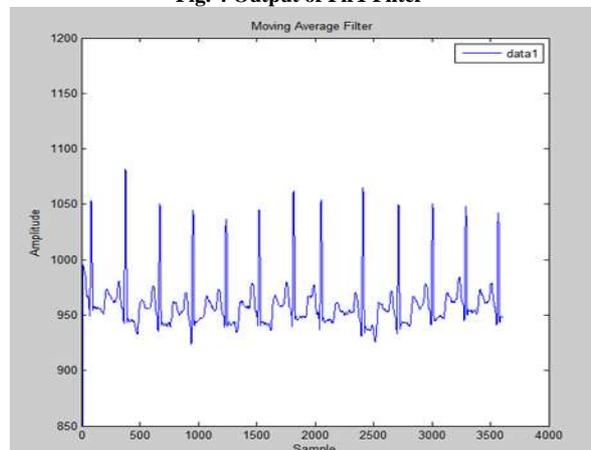


Fig. 6 Output of Moving Average Filter

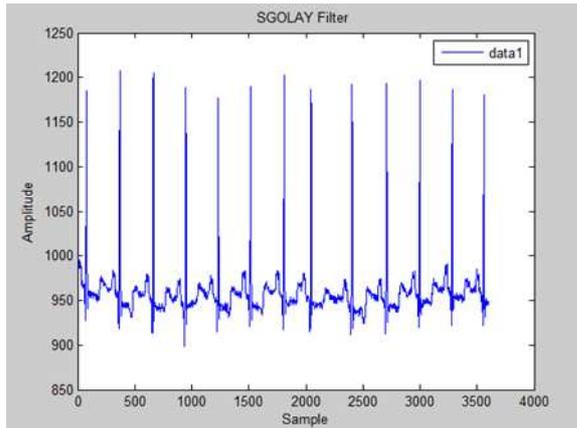


Fig. 7 Output of sgolay Filter

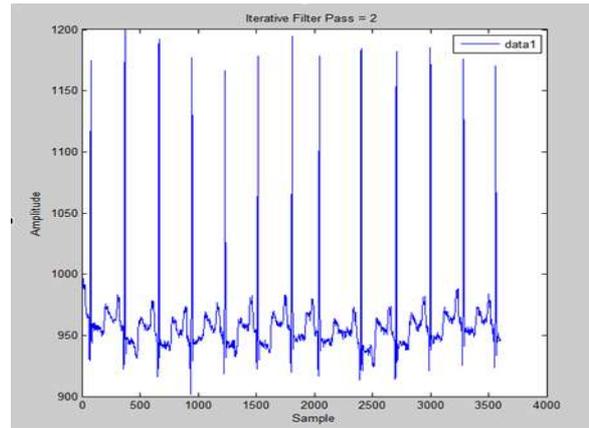


Fig. 8 Output of Iterative Filter with Pass = 2

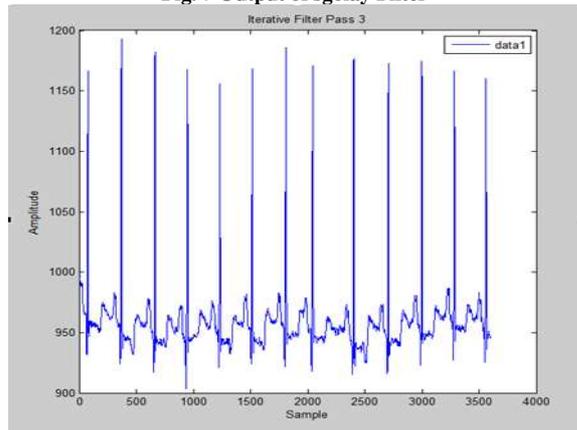


Fig. 9 Output of Iterative Filter with Pass = 3

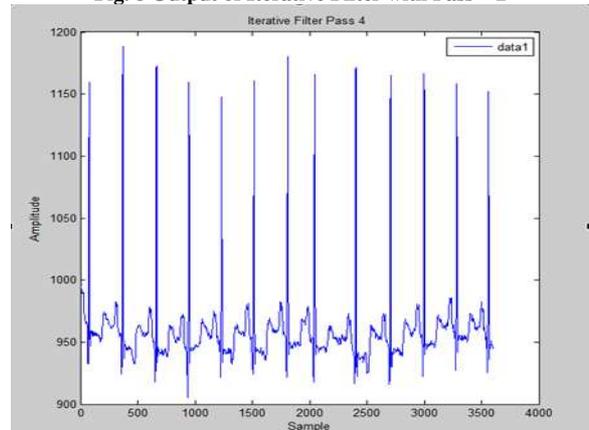


Fig. 10 Output of Iterative Filter with Pass = 4

**RESULT AND DISCUSSION**

Table -1 gives the comparison of filters based on the Signal to Noise Ratio (SNR) at various noise levels. Table -2 gives the comparison based on RMSE.

**Table -1 Comparison of filters based on SNR**

Filters	SNR ratio				
	Noise Level				
	at 5%	at 10%	at 20%	at 30%	at 50%
Noisy Signal	65.1096	58.8277	52.7384	49.1774	44.9506
Butterworth Filter	32.6704	32.6586	32.6247	32.5853	32.4241
Median Filter	33.992	33.9989	33.9362	33.875	33.6893
fir1 Filter	33.1802	33.1696	33.1418	33.1153	32.9812
Gaussian Filter	24.2584	24.2546	24.2485	24.2662	24.2157
Moving Average Filter	26.0154	26.0118	26.0053	26.0174	25.9632
Sgolay Filter	56.9968	53.366	51.4261	49.2161	45.9479
Iterative Filter Pass 2	56.5199	56.4297	56.0832	55.557	54.1704
Iterative Filter Pass 3	57.7443	57.7194	57.631	57.5334	57.0862
Iterative Filter Pass 4	58.846	58.8205	58.7092	58.5671	57.9804

**Table -2 Comparison of filters based on RMSE**

Filters	RMSE	NormRMSE	CV(RMSE)
Butterworth Filter	22.37484972	0.020706	0.023317
Median Filter	19.15003394	0.130539	0.019992
fir1 Filter	21.09264801	0.022098	0.021981
Gaussian Filter	58.76223277	0.050029	0.061338
Moving Average Filter	47.97395127	0.047232	0.050072
Sgolay Filter	2.282235746	0.007386	0.002377
Iterative Filter Pass 2	3.346953839	0.011194	0.003486
Iterative Filter Pass 3	4.481852296	0.015492	0.004669
Iterative Filter Pass 4	5.478083607	0.019296	0.005706

Table -3 Comparison of filters based on Minima, Maxima, Mean and Median

Filters	Min	Max	Mean	Median
Original Signal	895	1216	960	955
Sgolay Filter	899	1208	960	955.1
Iterative Filter Pass 2	901	1200	960	955
Iterative Filter Pass 3	903.7	1193	960	955.1
Iterative Filter Pass 4	905.1	1189	960	955.1

The experimental results show that sgolay and Iterative filters gives the best results for every criterion. Hence further discussion would revolve around only these filters.

Table -3 contains the values for maxima, minima, mean and median of the resultant signal after filtration for various signals. Data from Table -3 and figures 7-10 show that there is a shift in the first maxima and minima in the resultant signal for all the filters. But Pass 2 Iterative Filter was good enough to retain the mean and median of the original signal.

Data from Table -1 show that the Signal-to-Noise Ratio gets better with every iteration. With the increase in the noise level, there is a considerable improvement in the SNR. At the same time, the performance based on the criteria in Table -2 supports sgolay filter that gives better performance in terms of NormRMSE and CV (RMSE). But the performance of Iterative filter with Pass 2 is comparable with that of sgolay.

### CONCLUSION

From the above discussion, it is clear that the Iterative filter gave better results in case of noisy environments when compared to all other filters. But the major drawback is that the first wave is battered after filtration. To avoid erroneous outcomes due to this issue, the window can be chosen in such a way that it excludes the first wave. It is also seen that the root mean square error increases considerably with iteration. Hence, in the trade-off between better SNR and least RMSE, a proper value for iteration has to be chosen that gives the optimum result.

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