Fault Diagnosis in Cantilever Beam Using Fuzzy Logic

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ABSTRACT

Fault diagnosis is one of the major issues in every field of engineering, which may cause failure of the entire system. Fault diagnosis detection methods have been considerably increased over the past few decades. A presence of fault in structural member introduces local flexibility which affects the vibration response of the structure. The response of the system depends upon the type of fault occurs with its location. The damage in the structure leads to changes in natural frequencies and mode shapes. Early detection of presence of damage can prevent the catastrophic failure of the structures by appropriately monitoring the response to the system. In the present investigation fuzzy logic technique has been used to determine the fault in terms of crack. Here the transverse surface of the crack is considered and analyzed using FEA and fuzzy logic system. Analytical study has been performed on the cantilever beam with single crack to obtain the vibration characteristics of the beam. Here author intend to introduce fuzzy logic technique for fault diagnosis using several fuzzy rules. Here the first three natural frequencies are obtained and that are considered as input to fuzzy system. It is observed that the fuzzy controller can predict the depths and locations accurately close to the finite element analysis.

Key words: Vibration, Fault, Natural Frequency, Fuzzy Logic Techniques

INTRODUCTION

Crack is the potential source of failure in the field of engineering. Crack diagnosis in vibrating structure has drawn a lot of attention in mechanical machines and in civil structures and aerospace engineering. In the recent years the era of researchers has motivated towards development of intelligent techniques for crack detection. Many techniques have been employed in the past for damage identification. Some of these are visual (e.g. Dye Penetrant Method) and other NDT uses sensors to detect local faults (e.g. eddy current, magnetic field, radiographs, acoustics and thermal fields). In this chapter fuzzy logic technique has been projected for localization and identification of crack.

Fuzzy logic (FL) is a multi-valued logic, which allows interim values to be defined between linguistic expressions like yes/no, high/low, true/false. A form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts. Superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth - the truth values between "completely true & completely false". Fuzzy logic has two different meanings as, in narrow sense: Fuzzy logic is a logical system, which is an extension of multi-valued logic. In a wider sense: Fuzzy logic (FL) is almost synonymous with the theory of fuzzy sets, a theory which relates to classes of objects with unsharp boundaries in which membership is a matter of degree.

A K Das et al [1] have discussed the influence of cracks to the dynamic behaviour of a cracked cantilever beam with rectangular cross section. Finite element analysis is being performed on the cracked structure to measure the vibration signature, which is subsequently used in the design of smart system based fuzzy logic in prediction of crack depths and locations following inverse problem approach. Huh et al [2] has proposed a new local damage detection method of damaged structures using the vibratory power estimated from accelerations measured on the beam structure. A damage index is newly defined by them based on the proposed local damage detection method and is applied to the identification of structural damage. Numerical simulation and experiment are conducted for a uniform beam to confirm the validity of the proposed method. In the experiments, they have considered the damage as an open crack such as a
slit inflicted on the top surface of the beam. Parhi and Choudhary [3] presented non-destructive method for the detection of crack in terms of crack depth and crack location with the consideration of natural frequency. The crack is analyzed using Fuzzy Logic System and Finite Element Analysis. Zadeh [4] introduced and briefly analysed the relevant properties of fuzzy sets, the notions of a fuzzy system and a fuzzy class of systems. The work constitutes a very preliminary attempt on introducing into system theory several concepts which provide a way of treating fuzziness in a quantitative manner. The paper closes with a section dealing with optimization under fuzzy constraints in which an approach to problems of this type is briefly sketched. Salam et al [5] has proposed a simplified formula for the stress correction factor in terms of the crack depth to the beam height ratio. They have used the proposed formula to examine the lateral vibration of a Euler-Bernoulli beam with a single edge open crack and compared the mode shapes for the cracked and undamaged beam to identify the crack parameters.

Agarwalla et al [6] uses the GA – Fuzzy controller for the identification of damage in steel cantilever beam in transverse direction subjected to natural vibration. Parhi and Choudhary [7] have analyzed the transverse surface crack using fuzzy logic system and finite element analysis. The fuzzy controller uses the hybrid membership functions (combination of triangular, trapezoidal and Gaussian) as input and trapezoidal membership functions as output. By using several fuzzy rules, the results obtained for crack depth and crack location in the Matlab Simulink environment and have been compared with the results obtained from finite element analysis. Tahaa et al [8] has introduced a method to improve pattern recognition and damage detection by supplementing intelligent health monitoring with used fuzzy inferences system. The Bayesian methodology is used to demarcate the levels of damage to developing the fuzzy system and is examined to provide damage identification using data obtained from finite element analysis for a pre-stressed concrete bridge. Wada et al [9] has proposed a fuzzy control method of triangular type membership functions using an image processing unit to control the level of granules inside a hopper. They stated that the image processing unit can be used as a detecting element and with the use of fuzzy reasoning methods good process responses were obtained. Parhi and Choudhary [10] describe a comprehensive review of various technical papers in the domain of crack detection in Beam-Like Structure. The various techniques discussed on the basis of dynamic analysis of Crack. The techniques mainly of fuzzy logic neural network, fuzzy system, hybrid neuro genetic algorithm, artificial neural network, artificial intelligence. Parhi [11] has developed a fuzzy inference based navigational control system for multiple robots working in a clummy environment. They have been designed to navigate in an environment without hitting any obstacles along with other robots. Zimmermann [12] has applied fuzzy linear programming approach to solving linear vector maximum problem. The solutions are obtained by fuzzy linear programming. These are found to be efficient solutions then the numerous models suggested solving the vector maximum problem.

FINITE ELEMENT FORMULATION

The free bending vibration of a Euler-Bernoulli beam of a constant rectangular cross section is given by the following differential equation as given in:

\[
\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{\rho \omega^2}{EI} \phi(x) = 0
\]  

(1)

By defining \( \beta^4 = \frac{\rho \omega^2}{EI} \), the equation is rearranged as a fourth order differential equation as

\[
\frac{\partial^4 \phi(x)}{\partial x^4} - \beta^4 \phi(x) = 0
\]  

(2)

The general solution to this equation is

\[
\phi(x) = A \sinh(\beta x) + B \cosh(\beta x) + C \sin(\beta x) + D \cos(\beta x)
\]  

(3)

Where \( A, B, C, D \) are constants and \( \beta \) is a frequency parameter. Adopting Hermitian shape functions, the stiffness matrix of the two-noded beam element without a crack is obtained using the standard integration based on the variation in flexural rigidity.

The element stiffness of the un cracked beam is given by

\[
[K_e] = \int [B(x)]^T [EI] [B(x)] \, dx
\]  

(4)

\[
[B(x)] = \{H_1(x), H_2(x), H_3(x), H_4(x)\}
\]  

(5)

Where \( [H_1(x), H_2(x), H_3(x), H_4(x)] \) is the Hermitian shape functions defined as

\[
H_1(x) = 1 - \frac{3x^2}{1^2} + \frac{2x^3}{1^3}, \quad H_2(x) = x - \frac{2x^2}{1^2} + \frac{x^3}{1^2}, \quad H_3(x) = 1 - \frac{3x^2}{2^2} - \frac{2x^3}{2^3}, \quad H_4(x) = -\frac{x^2}{1^2} + \frac{x^3}{1^2}
\]  

(6)

Assuming the beam rigidity EI is constant and is given by EI\( q \) within the element, and then the element stiffness is
\[
[K^e] = \frac{EI_1}{L^2} \begin{bmatrix} 12 & 61 & -12 & 61 \\ 61 & 41^2 & -61 & 21^2 \\ -12 & -61 & 12 & -61 \\ 61 & 21^2 & -61 & 41^2 \end{bmatrix}
\] (7)

\[
[K_{c}^e] = [K^e] - [K_c]
\] (8)

Here, \([K_{c}^e]\) = Stiffness matrix of the cracked element, \([K^e]\) = Element stiffness matrix, \([K_c]\) = Reduction in stiffness matrix due to the crack.

According to Peng, the matrix \([K_c]\) is

\[
[K_c] = \begin{bmatrix} k_{11} & -k_{12} & k_{14} \\ k_{12} & k_{22} & -k_{12} & k_{24} \\ -k_{11} & -k_{12} & k_{11} & -k_{14} \\ k_{14} & k_{24} & -k_{14} & k_{44} \end{bmatrix}
\] (9)

Where, \(k_{11} = \frac{12EI_0 - I_c}{L^4} \left[ \frac{21L^4}{L^2} + 31c \left( \frac{2L_1}{L^2} - 1 \right) \right] \) (11)

\[
K_{12} = \frac{12EI_0 - I_c}{L^3} \left[ \frac{13L^3}{L^2} + 1c \left( 2 - \frac{7L_1}{L} + \frac{6L_1^2}{L^2} \right) \right] \)
\]

\[
K_{14} = \frac{12EI_0 - I_c}{L^3} \left[ \frac{31L^3}{L^2} + 1c \left( 2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2} \right) \right] \)
\]

\[
K_{24} = \frac{12EI_0 - I_c}{L^3} \left[ \frac{31L^3}{L^2} + 1c \left( 2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2} \right) \right] \)
\]

Here, \(I_c = 1.5W, L=\)Total length of the beam, \(L_1 = \)Distance between the left node and crack

\(I_0 = \frac{BW^3}{12} \) = Moment of inertia of the beam cross section, \(I_c = \frac{B(W-a)^3}{12} \) = Moment of inertia of the beam with crack.

It is supposed that the crack does not affect the mass distribution of the beam. Therefore, the consistent mass matrix of the beam element can be formulated directly as

\[
[M^e] = \int_0^1 \rho A[H(x)]^T[H(x)] dx
\] (10)

\[
[M^e] = \frac{\rho A}{420} \begin{bmatrix} 156 & 221 & 54 & -131 \\ 221 & 41^2 & 131 - 31^2 \\ 54 & 131 & 156 - 221 \\ -131 - 31^2 & -221 & 41^2 \end{bmatrix}
\] (11)

The natural frequency then can be calculated from the relation.

\[
[-\omega^2 [M] + [K]] (q) = 0
\] (12)

Where, \(q\) = displacement vector of the beam

**ANALYSIS OF FUZZY MECHANISM USED FOR CRACK DETECTION**

The fuzzy controller has been developed (as shown in Fig.:1) where there are 3 inputs and 2 outputs parameter.

![Fig. 1 Schematic Diagram of Fuzzy Inference System](image-url)
The linguistic term used for the inputs are as follows;

- Relative first natural frequency = FNF
- Relative second natural frequency = SNF
- Relative third natural frequency = TNF
- Relative crack depth = RCD
- Relative crack location = RCL

The fuzzy models developed in the current analysis, based on triangular, Gaussian and trapezoidal membership functions have got three or six input parameters and two or four output parameters. The pictorial view of the triangular membership, Gaussian membership, trapezoidal membership fuzzy models are shown in Fig. 2(a), Fig. 2(b) and Fig. 2(c) respectively.

Based on the above fuzzy subset the fuzzy rules are defined in a general form as follows: If (FNF is FNFi and SNF is SNFj and TNF is TNFk) then (CD is CDijk and CL is CLijk) Where i= 1 to 9, j=1 to 9, k=1 to 9

\[(13)\]

Because of “FNF”, “SNF”, “TNF” have 9 membership functions each.

From the above expression (11), two set of rules can be written

- If (FNF is FNFi and SNF is SNFj and TNF is TNFk) then CD is CDijk
- If (FNF is FNFi and SNF is SNFj and TNF is TNFk) then CL is CLijk

\[(14a)\]
\[(14b)\]

According to the usual Fuzzy logic control method (Das and Parhi [1]), a factor Wijk is defined for the rules as follows:

\[W_{ijk} = \mu_{fnfi}(freq_i) \Lambda \mu_{snfj}(freq_j) \Lambda \mu_{tnfi}(freq_k)\]

Where freqi, freqj and freqk are the first, second and third natural frequency of the cantilever beam with crack respectively. By applying composition rule of interference (Das and Parhi [1,13]) the membership values of the relative crack location and relative crack depth (location) CL.

\[\mu_{rclijk}(location) = W_ijk \Lambda \mu_{rclijk}(location)\]
\[\mu_{rclijk}(depth) = W_ijk \Lambda \mu_{rclijk}(depth)\]

As; \[\mu_{rclijk}(location)\] and \[\mu_{rclijk}(depth)\]

The overall conclusion by combining the output of all the fuzzy can be written as follows:

\[\mu_{rclijk}(location) = \mu_{rcl111}(location) \vee \ldots \vee \mu_{rclijk}(location)\]
\[\mu_{rclijk}(depth) = \mu_{rcl111}(depth) \vee \ldots \vee \mu_{rclijk}(depth)\]

\[(15a)\]
\[(15b)\]

The crisp values of relative crack location and relative crack depth are computed using the centre of gravity method (Das and Parhi [1, 13]) as:

\[\text{Relative crack location} = \frac{\int \text{location}. \mu_{rcl(location)}.d(location)}{\int \mu_{rcl(location)}.d(location)}\]
\[\text{Relative crack depth} = \frac{\int \text{depth}. \mu_{rcl(depth)}.d(depth)}{\int \mu_{rcl(depth)}.d(depth)}\]
RESULTS AND DISCUSSION

The fuzzy controller has been designed using three types of membership functions, i.e. Triangular, Trapezoidal and Gaussian membership function. The linguistic terms used for the fuzzy membership function have been specified in Table -1. The fuzzy rules being used for the fuzzy inference system are specified in the Table - 2. Out of several hundreds of fuzzy rules only twenty-four fuzzy rules have been indicated in the table. Fig. 3 to Fig. 5 shows the operation of fuzzy inference system to exhibit the fuzzy results after defuzzification when rule 2 and 15 of the Table -2 are activated for triangular, trapezoidal, Gaussian and hybrid membership functions respectively. The comparison of the results obtained from theoretical and the fuzzy controller of triangular membership function, fuzzy controller with trapezoidal membership function, fuzzy controller with Gaussian membership function are presented in Table - 3. Table - 4 shows the comparisons of natural frequencies of the beam with various crack depth and location by theoretical, ANSYS and experimental method. Graph: 1 to Graph: 4 show the natural frequencies obtained by experimental method (FFT analysis).

### Table -1 Linguistic Term used for Fuzzy Membership Functions

<table>
<thead>
<tr>
<th>Name of the Membership functions</th>
<th>Linguistic terms</th>
<th>Description and range of the linguistic terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1F1,L1F2,L1F3,L1F4</td>
<td>tnf 1t04</td>
<td>Low ranges of relative natural frequency for first mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>M1F1,M1F2,M1F3</td>
<td>Pnf 5t07</td>
<td>Medium ranges of relative natural frequency for first mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>H1F1,H1F2,H1F3,H1F4</td>
<td>tnfs0011</td>
<td>Higher ranges of relative natural frequency for first mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>L2F1,L2F2,L2F3,L2F4</td>
<td>snf1t04</td>
<td>Low ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>M2F1,M2F2,M2F3</td>
<td>snfs5t07</td>
<td>Medium ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>H2F1,H2F2,H2F3,H2F4</td>
<td>snfs0011</td>
<td>Higher ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>L3F1,L3F2,L3F3,L3F4</td>
<td>tnfs0011</td>
<td>Higher ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>M3F1,M3F2,M3F3</td>
<td>tnfs5t07</td>
<td>Medium ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>H3F1,H3F2,H3F3,H3F4</td>
<td>tnfs0011</td>
<td>Higher ranges of relative natural frequency for second mode of vibration in ascending order respectively.</td>
</tr>
<tr>
<td>SD1,SD2,SD3,SD4</td>
<td>dcd1t04</td>
<td>Small ranges relative crack depth in ascending order respectively.</td>
</tr>
<tr>
<td>MD1,MD2,MD3</td>
<td>dcd5t07</td>
<td>Medium ranges relative crack depth in ascending order respectively.</td>
</tr>
<tr>
<td>LD1,LD2,LD3,LD4</td>
<td>dcd8s011</td>
<td>Larger ranges of relative crack depth in ascending order respectively.</td>
</tr>
<tr>
<td>SL1,SL2,SL3,SL4</td>
<td>dcd1t04</td>
<td>Small ranges of relative crack depth in ascending order respectively.</td>
</tr>
<tr>
<td>ML1,ML2,ML3</td>
<td>dcd5s07</td>
<td>Medium ranges of relative crack location in ascending order respectively.</td>
</tr>
<tr>
<td>BL1,BL2,BL3,BL4</td>
<td>dcd8s011</td>
<td>Bigger ranges of relative crack location in ascending order.</td>
</tr>
</tbody>
</table>

### Table -2 Fuzzy Rules for Fuzzy Inference System

<table>
<thead>
<tr>
<th>S. No</th>
<th>Some Examples of Fuzzy rule used in the Fuzzy Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>If tnf is L1F1, snf is L2F1, tnf is L3F1 then dcd is SD1 and rel is SL1</td>
</tr>
<tr>
<td>2</td>
<td>If tnf is L1F1, snf is L2F2, tnf is L3F2 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>3</td>
<td>If tnf is L1F1, snf is L2F2, tnf is L3F3 then dcd is SD1 and rel is SL2</td>
</tr>
<tr>
<td>4</td>
<td>If tnf is M1F1, snf is M2F1, tnf is M3F1 then dcd is SD1 and rel is SL1</td>
</tr>
<tr>
<td>5</td>
<td>If tnf is M1F1, snf is M2F2, tnf is M3F2 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>6</td>
<td>If tnf is M1F1, snf is M2F2, tnf is M3F3 then dcd is SD1 and rel is SL2</td>
</tr>
<tr>
<td>7</td>
<td>If tnf is M1F2, snf is M2F1, tnf is M3F1 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>8</td>
<td>If tnf is M1F2, snf is M2F2, tnf is M3F2 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>9</td>
<td>If tnf is M1F2, snf is M2F2, tnf is M3F3 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>10</td>
<td>If tnf is M2F1, snf is M2F2, tnf is M3F1 then dcd is SD1 and rel is SL1</td>
</tr>
<tr>
<td>11</td>
<td>If tnf is L1F2, snf is L2F1, tnf is L3F1 then dcd is SD2 and rel is SL1</td>
</tr>
<tr>
<td>12</td>
<td>If tnf is L1F2, snf is L2F2, tnf is L3F2 then dcd is SD2 and rel is SL2</td>
</tr>
<tr>
<td>13</td>
<td>If tnf is L1F2, snf is L2F3, tnf is L3F3 then dcd is SD2 and rel is SL3</td>
</tr>
<tr>
<td>14</td>
<td>If tnf is L1F3, snf is L2F1, tnf is L3F1 then dcd is SD3 and rel is SL1</td>
</tr>
<tr>
<td>15</td>
<td>If tnf is L1F3, snf is L2F2, tnf is L3F2 then dcd is SD3 and rel is SL3</td>
</tr>
<tr>
<td>16</td>
<td>If tnf is L1F3, snf is L2F3, tnf is L3F3 then dcd is SD3 and rel is SL3</td>
</tr>
<tr>
<td>17</td>
<td>If tnf is H1F1, snf is H2F1, tnf is H3F1 then dcd is LD1 and rel is BL1</td>
</tr>
<tr>
<td>18</td>
<td>If tnf is H1F1, snf is H2F2, tnf is H3F2 then dcd is LD2 and rel is BL2</td>
</tr>
<tr>
<td>19</td>
<td>If tnf is H1F1, snf is H2F3, tnf is H3F3 then dcd is LD1 and rel is BL2</td>
</tr>
<tr>
<td>20</td>
<td>If tnf is H1F2, snf is H2F1, tnf is H3F1 then dcd is LD2 and rel is BL1</td>
</tr>
<tr>
<td>21</td>
<td>If tnf is H1F2, snf is H2F2, tnf is H3F2 then dcd is LD2 and rel is BL3</td>
</tr>
<tr>
<td>22</td>
<td>If tnf is H1F2, snf is H2F3, tnf is H3F3 then dcd is LD3 and rel is BL1</td>
</tr>
<tr>
<td>23</td>
<td>If tnf is H2F1, snf is H2F2, tnf is H3F2 then dcd is LD1 and rel is BL3</td>
</tr>
<tr>
<td>24</td>
<td>If tnf is H2F3, snf is H2F3, tnf is H3F3 then dcd is LD3 and rel is BL3</td>
</tr>
</tbody>
</table>
Inputs for Trapezoidal Membership Function

Rule No. 2 of Table -2 is activated

Rule No. 15 of Table -2 is activated

Outputs Obtained from Triangular Membership Function

Relative Crack Depth

Relative Crack Location

Fig. 3 Resultant Values of Relative Crack Depth and Relative Crack Location of Triangular Membership Function When Rules 2 And 15 of Table -2 Are Activated

Inputs for Trapezoidal Membership Function

Rule No. 2 of Table -2 is activated

Rule No.15 of Table -2 is activated
Outputs Obtained from Trapezoidal Membership Function

Relative Crack Depth
Relative Crack Location
Fig. 4 Resultant Values of Relative Crack Depth and Relative Crack Location of Trapezoidal Membership Function When Rules 2 and 15 of Table -2 Are Activated

Inputs for Gaussian Membership Function
Rule No. 2 of Table -2 is activated
Rule No. 15 of Table -2 is activated

Outputs Obtained from Gaussian Membership Function

Relative Crack Depth
Relative Crack Location
Fig. 5 Resultant Values of Relative Crack Depth and Relative Crack Location of Gaussian Membership Function When Rules 2 and 15 of Table -2 Are Activated

Graph: 1 Natural Frequency of the Crack Beam at 1@100 (mm):
Graph: 2 Natural Frequency of the Crack Beam at 1@200 (mm):

Graph: 3 Natural Frequency of the Crack Beam at 2@100 (mm)

Graph: 4 Natural Frequency of the Crack Beam at 2@200 (mm)
Table 3: Comparisons of Results between Theoretical Analysis and Different Fuzzy Controller Analysis

<table>
<thead>
<tr>
<th></th>
<th>First Natural Frequency</th>
<th>Second Natural Frequency</th>
<th>Third Natural Frequency</th>
<th>Theoretical</th>
<th>Triangular Fuzzy Controller</th>
<th>Trapezoidal Fuzzy Controller</th>
<th>Gaussian Fuzzy Controller</th>
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<tr>
<td></td>
<td>fnf</td>
<td>snf</td>
<td>tnf</td>
<td>red</td>
<td>rel</td>
<td>red</td>
<td>rel</td>
</tr>
<tr>
<td>Relative crack depth rcd</td>
<td>0.2</td>
<td>0.25</td>
<td>0.213</td>
<td>0.261</td>
<td>0.212</td>
<td>0.258</td>
<td>0.21</td>
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<tr>
<td>Relative crack location rcl</td>
<td>0.5</td>
<td>0.25</td>
<td>0.246</td>
<td>0.587</td>
<td>0.237</td>
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<td></td>
<td>47.864</td>
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<td>815.502</td>
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<td></td>
<td>48.197</td>
<td>290.257</td>
<td>821.870</td>
<td></td>
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<tr>
<td></td>
<td>46.455</td>
<td>293.842</td>
<td>809.819</td>
<td>0.2</td>
<td>0.25</td>
<td>0.423</td>
<td>0.259</td>
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<tr>
<td></td>
<td>48.991</td>
<td>289.865</td>
<td>827.046</td>
<td>0.4</td>
<td>0.5</td>
<td>0.425</td>
<td>0.537</td>
</tr>
</tbody>
</table>

Natural Frequencies of Cantilever Beam using ANSYS

For crack 1mm depth at 100 mm length

![Fig. 6 (a) First Natural Frequency](image1)

![Fig. 6 (b) Second Natural Frequency](image2)

![Fig. 6 (c) Third Natural Frequency](image3)

For crack 1mm depth at 200 mm length

![Fig. 7 (a) First Natural Frequency](image4)

![Fig. 7 (b) Second Natural Frequency](image5)

![Fig. 7 (c) Third Natural Frequency](image6)
For crack 2mm depth at 100 mm length

For crack 2mm depth at 200 mm length

Fig. 8 (a) First Natural Frequency

Fig. 9 (a) First Natural Frequency

Fig. 8 (b) Second Natural Frequency

Fig. 9 (b) Second Natural Frequency

Fig. 8 (c) Third Natural Frequency

Fig. 9 (c) Third Natural Frequency

Table - 4 Natural Frequencies of Cantilever Beam

<table>
<thead>
<tr>
<th>Frequency</th>
<th>First Natural Frequency</th>
<th>Second Natural Frequency</th>
<th>Third Natural Frequency</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>ANSYS</td>
<td>Experimental</td>
</tr>
<tr>
<td>Crack (mm) 1@100</td>
<td>47.864</td>
<td>46.3781</td>
<td>47.104</td>
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<tr>
<td>Crack (mm) 1@200</td>
<td>48.197</td>
<td>46.5864</td>
<td>47.408</td>
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<tr>
<td>Crack (mm) 2@100</td>
<td>46.455</td>
<td>45.709</td>
<td>46.007</td>
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<tr>
<td>Crack (mm) 2@200</td>
<td>48.991</td>
<td>47.0505</td>
<td>48.347</td>
</tr>
</tbody>
</table>

Chaudhari and Patil  
CONCLUSION

The fuzzy controller has been designed using Triangular, Trapezoidal and Gaussian membership function. A fuzzy controller uses three natural frequencies as inputs whereas the crack depth and crack location as output. It has been observed that the natural frequencies of the beam are changing into change in crack depth and crack location. The predicted results from fuzzy controllers of crack location and crack depth are compared with the theoretical results. It is observed from the Table -3 that the results obtained from Gaussian membership function fuzzy controller predict more accurate result in comparison to other three controllers. The cracks depth and crack location obtained from the Gaussian membership function are nearness to the theoretical value.

REFERENCES