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Solution of the Number of Servers Pivot on Batch Arrivals

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ABSTRACT

In this paper, investigate the steady state behaviour of a queueing system by assuming that the numbers of server's change depending on the certain queue length. In the system arrivals occurs in batches. In the case of two servers, it is assumed that the service abilities of additional servers are differ. Whenever the queue length in front of the first regular server reaches a specific length then additional server will be added instantly. The mean queue lengths of customers are derived explicitly.

Keywords: Queue-dependent servers, Heterogeneous, Bulk queues, Average queue length.

INTRODUCTION

In many real life situations, it is not abnormal to consider that additional servers are provided to reduce congestion. When many customers are waiting for service, for example, in the post office, banks, at the registers of super markets, ticket offices of stations, etc., the decision-maker often provides additional server to reduce the congestion i.e. waiting line. The main purpose of this study was to establish an evaluation model as a reference for additional service facility required. In real life, it is normal to see all sorts of queues for service such as queues of cars waiting to be filled up at a petrol station and soon- these are situations for the implementation of additional server facility in queueing theory. In practical procedure of planning for additional server, decision makers may base their plain on the first server queue length and to decide the additional server for facilities to reduce the waiting line. It then constructs a queue dependent server queueing system. The congestion of queueing system initiates another server whenever the number of customers in system reaches a certain length and turns on the second server for service facilities. A lot of queueing problems where the service is through parallel or multi channels have been solved by various authors.

In particular Singh [4] has analysed a queueing system where the number of servers changes depending on the queue length. When a procession of customers waiting in front of the first server reaches a certain queue length i.e. finite capacity, the second server joins the system and begins serving. That is, a queueing system with two servers is formed. Cases where the service abilities of the servers are identical and different have been explained. In a procession of customers waiting in front of two servers, it is assumed that infinite queuing is applicable. Garg and Singh [5] tackled the queue-dependent problem to obtain the optimal queue length under a cost structure so as to maximize the total profit. Yamashiro [9] extended the work of Singh [4] and Garg and Singh [5] by taking state dependent servers. Jain et al. [10] analysed M/M/C/K/N machine repair problem with balking, reneging, spares and additional repairman. Jain and Singh [7] incorporated additional servers for M/M/m queue with balking and reneging. Wang and Tai [6] analysed the finite source M/M/3 queueing system with the service abilities of each server are different. Murari [11] obtain the steady state probabilities of queueing problems with variable number of service channels and some others books [1-3]. Garg *et al* [12] analysed the one server reach their capacity in the queueing system. Also we ref the authors papers [15-17]. The purpose of this paper, we derive the steady state solution for the finite source queueing system with queue dependent servers (batch arrivals) which are used to obtain the various system characteristics such as the average number of customers in the system, the average number of customers in the queue length.

MODEL DESCRIPTION

The queueing system investigated in this paper is based on the following assumptions: -

- The customers occur in batches in the system of variable size with maximum limits K and arrivals follow Poisson distribution with arrival rate λ.
- The queue discipline is first-come, first-served (FCFS). The services times are assumed to be distributed according to an exponential distribution with the following probability density function:

$$s(t) = \mu \exp(-\mu t), t \geq 0, \mu \geq 0 \tag{1}$$

- The service facility consists of two service channels i.e. one regular and one additional. Whenever the queue length in front of the regular service channel reaches a specific queue length (the number of customers waiting including those being served). The additional service channel operates instantaneously on an arrival when there are $K<2^{-1}N$ customers in the queue and not operating when the queue length again reduces to N.

STEADY STATE RESULTS

Let $P_n \equiv$ the steady-state probability that there are n customers in the system at any time, where $n = 1, 2, 3, ..., K, ...N$.

The steady-state equations for the finite source queueing system with queue-dependent servers are given by

$$\mu P_1 = \lambda P_0 \tag{2}$$

$$\mu P_n = (\lambda + \mu) P_{n-1} - \lambda \sum_{j=1}^{K} M_j P_{n-(j+1)}, \quad n = 2, 3, .., N \tag{3}$$

$$2\mu P_{N+1} = (\lambda + \mu) P_N - \lambda \sum_{j=1}^{K} M_j P_{N-j} \tag{4}$$

Where $\sum_{j=1}^{K} M_j = 1, M_j$ is the probability that a batch arrivals consists of exactly j customers $j = 1, 2, .., K$ Solving equations (2)-(4) recursively, we obtain the steady-state solutions P_n, respectively

$$P_n = \begin{cases} \sum_{j=1}^{K} \gamma_j \pi_j^n P_0, & 1 \leq n \leq K+1 \\ \sum_{j=1}^{K} A_j \pi_j^n P_0, & K+2 \leq n \leq N-K \\ \sum_{j=1}^{K} \alpha_j \pi_j^n P_0, & N-(K-1) \leq n \leq N \\ \sum_{j=1}^{K} A_j \pi_j^n P_0, & N+2 \leq n \end{cases} \tag{5}$$

$$\rho = \lambda \mu^{-1}$$

$$\gamma_j = \frac{\pi_j^{K-1}}{\prod_{\substack{r=1 \\ r \neq j}}^{K} (\pi_j - \pi_r)}, 1 \leq j \leq K$$

$$A_j = \frac{\sum_{g=1}^{K} \gamma_g \left[\rho \sum_{i=1}^{K} \sum_{T=0}^{i-1} M_{K-T} \pi_j^{T-i} \pi_g^i - \pi_g^{K+1} \right]}{(\pi_j - \pi_j^2) \prod_{\substack{r=1 \\ r \neq j}}^{K} (\pi_j - \pi_r)}, 1 \leq j \leq K$$

$$\alpha_j = \frac{\sum_{g=1}^{K} A_g \left[\rho \sum_{i=1}^{K} \sum_{T=0}^{i-1} M_{K-T} \pi_j^{T-i} \pi_g^{N-K+i} - \pi_g^{N+1} \right]}{2(\pi_j^{N-K+1} - \pi_j^{2N-2K+2}) \prod_{\substack{r=1 \\ r \neq j}}^{K} (\pi_j - \pi_r)}, 1 \le j \le K$$

Where

$\gamma_j, \pi_j; 1 \le j \le K$ are respectively the primitive roots of unity of the algebraic equations.

$$Z^{K+1} - \left(1 + 2\lambda\mu^{-1}\right) Z^K + 2\lambda\mu^{-1} \left(M_1 Z^{K-1} + M_2 Z^{K-2} + M_3 Z^{K-3} + \dots + M_K \right) = 0 \tag{6}$$

Since $\sum_{n=0}^{\infty} P_n = 1$, every π_j must be lie within the unit circle and By Rouche's theorem it can be easily shown that this condition holds provided $K\lambda\mu^{-1} < 2$. Now assuming that this condition is satisfied and using the normal condition that $\sum_{n=0}^{\infty} P_n = 1$. Final we get

$$P_0 = \frac{1}{\left[1 + \sum_{j=1}^{K} \frac{\gamma_g (\pi_j^1 - \pi_j^{N+1})}{1 - \pi_j} + \frac{A_j \pi_j^{N+1}}{1 - \pi_j^1} \right]} \tag{7}$$

Average Queue Length

Assume L_2 represent the average queue length,

$$L_2 = \sum_{n=0}^{N} n P_n + \sum_{n=N+1}^{\infty} n P_n \tag{8}$$

Substituting the value of P_n, we have

$$L_2 = \sum_{j=1}^{K} \left[\frac{A_j \pi_j - (N+1) A_j \pi_j^{N+1} + N A_j \pi_j^{N+1}}{\left(1 - \pi_j\right)^2} + \frac{(N+1)\alpha_j \pi_j^{N+1} - N\alpha_j \pi_j^{N+2}}{\left(1 - \pi_j^N\right)^2} \right] P_0 \tag{9}$$

Particular Case

When the arrival is occurring single the situation corresponds to Queue-dependent servers by Singh [4]. When $\mathrm{K=1,\ M_1=1, M_j = 0}\ 2 \le \mathrm{j} \le \mathrm{K}$ then we have

$$L_2 = \frac{1}{\left(2 - \rho - \rho^{N+1}\right)} \left[\frac{1}{(1-\rho)} \left\{ \rho(2-\rho) - \left(2\rho^{N+1} - \rho^{N+2}\right)\left(N - \rho N + 1\right) \right\} + \frac{\left(\rho^{N+1} - \rho^{N+2}\right)\left(2N - \rho N + 2\right)}{\left(2 - \rho\right)} \right] \tag{10}$$

These results are in conformity with that of Singh [4].

CONCLUSION

This paper, find the queue length in front of the first regular server reaches a specific length then additional server will be added instantly. Also, find mean queue lengths of customers.

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